

Analisi di Immagini e Video (Computer Vision)

Giuseppe Manco

Outline

- Image Processing avanzato
 - Edge detection
 - Fourier Transform

Crediti

- Slides adattate da vari corsi
 - Analisi di Immagini (F. Angiulli) – Unical
 - Intro to Computer Vision (J. Tompkin) – CS Brown Edu
 - Computer Vision (I. Gkioulekas), CS CMU Edu

Analisi di Fourier

Trasformata di Fourier

$$A \sin(\omega x + \phi)$$

Ogni segnale periodico è una combinazione di queste componenti

Trasformata di Fourier

$$A \sin(\omega x + \phi)$$

Ampiezza

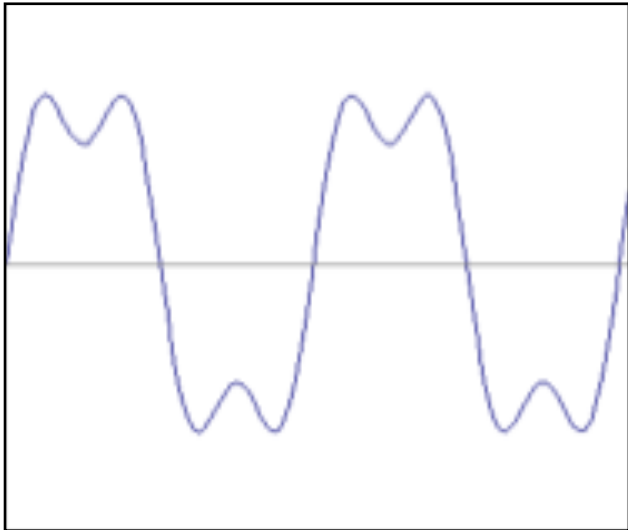
Sinusoide

Frequenza angolare

Variabile

Fase

Esempio

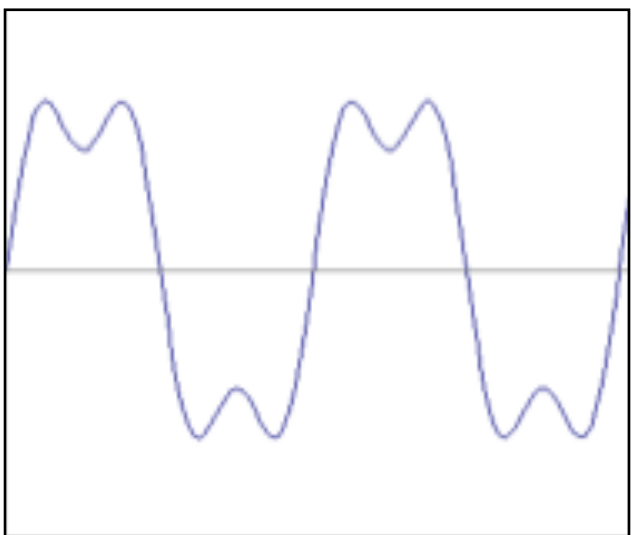


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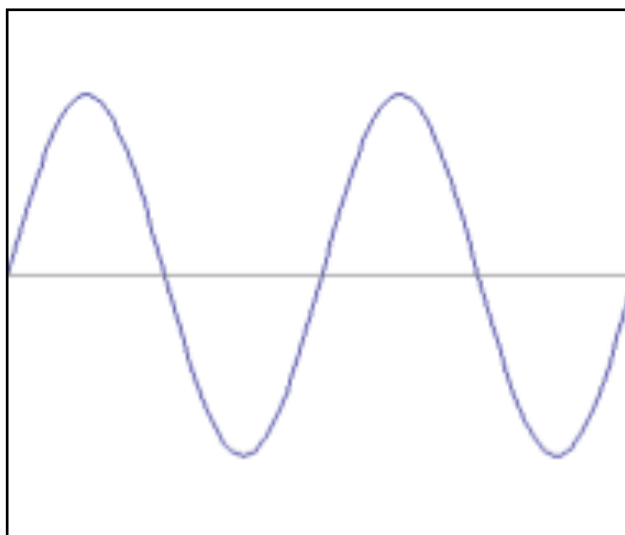
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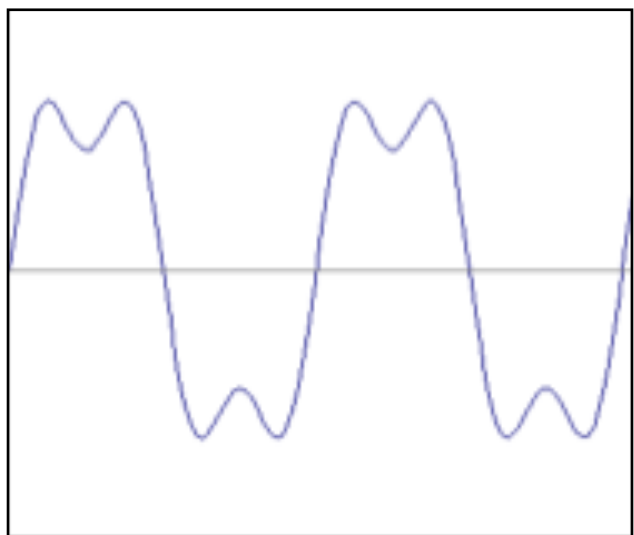
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$\sin(2\pi x)$

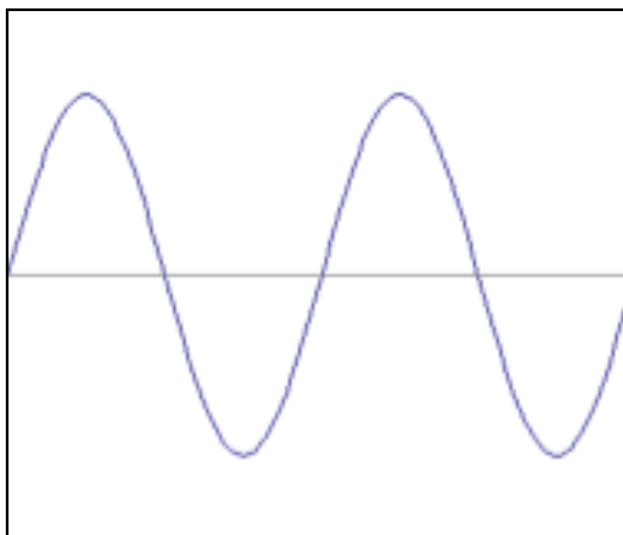
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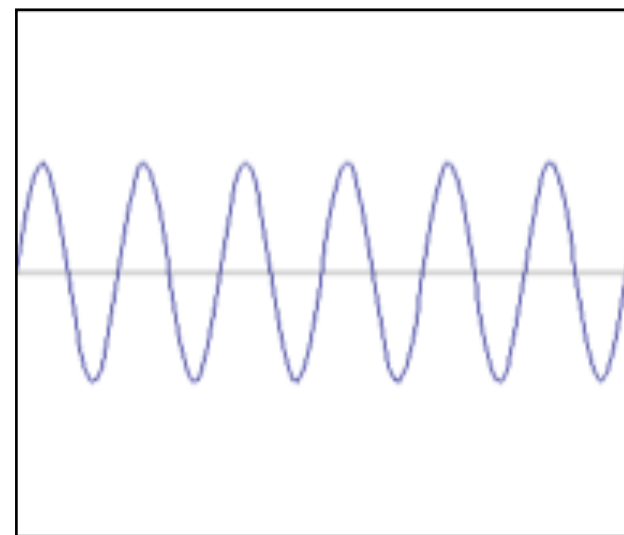
$$f(x) = \sin(2\pi x) + \frac{1}{3} \sin(2\pi 3x)$$

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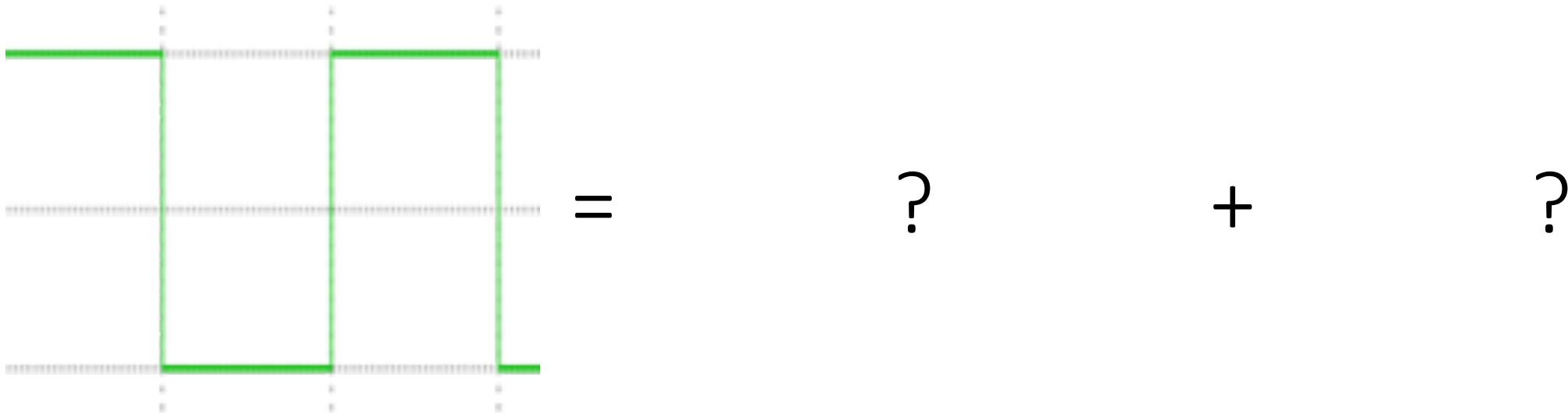
$$\sin(2\pi x)$$

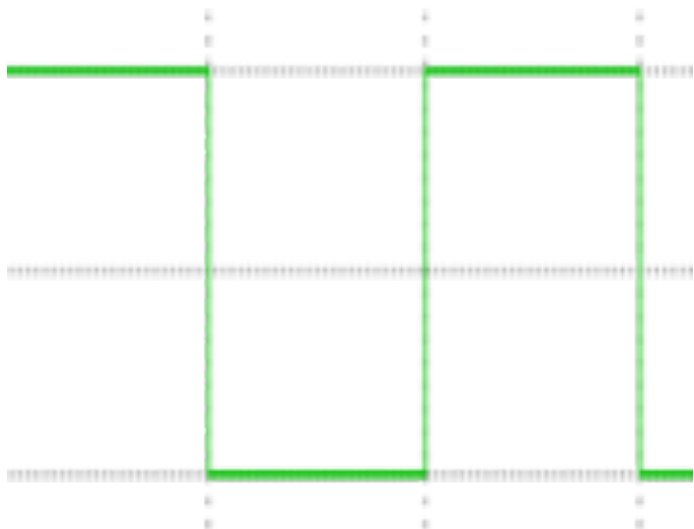
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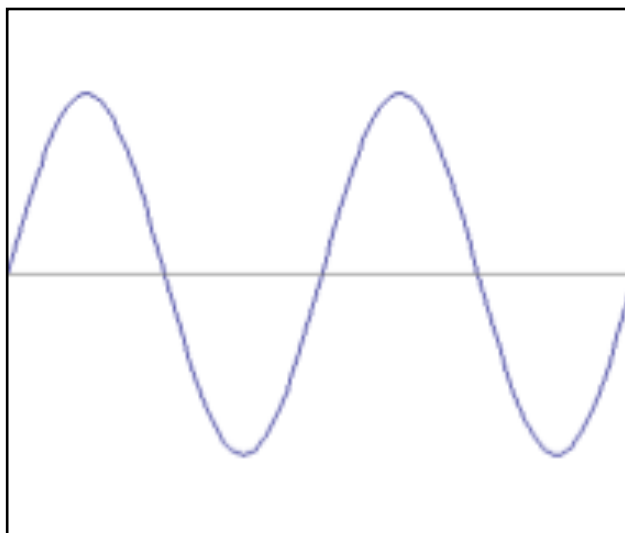
$$\frac{1}{3} \sin(2\pi 3x)$$

Esempio

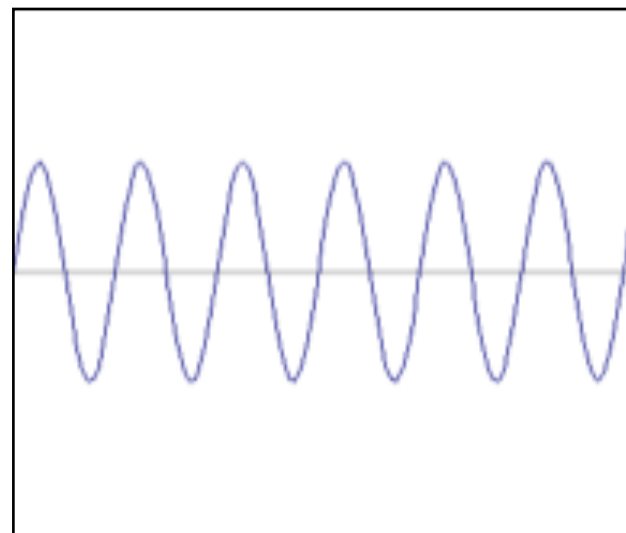




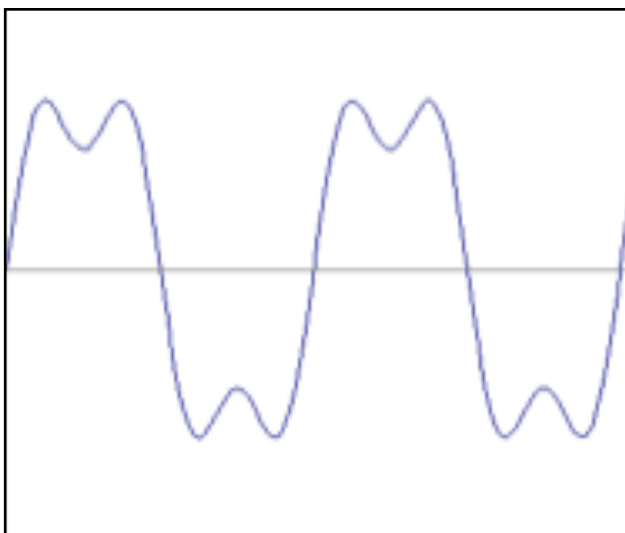
\approx

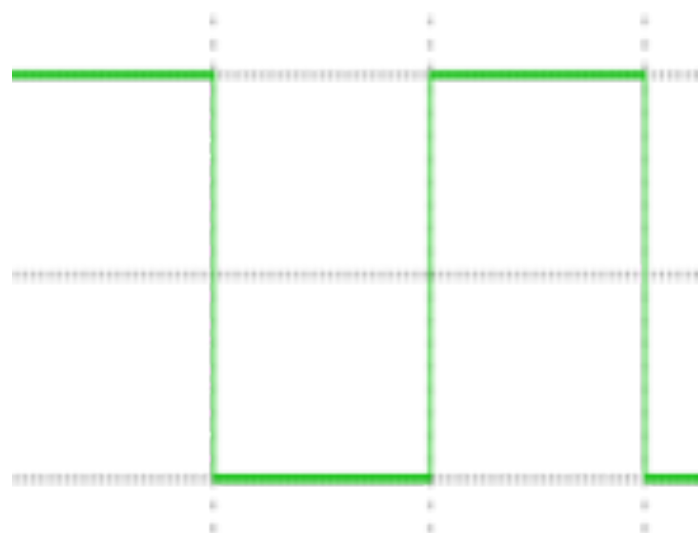


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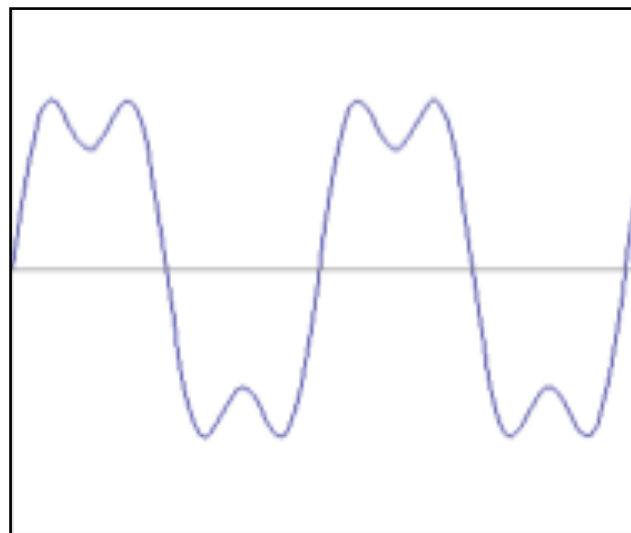


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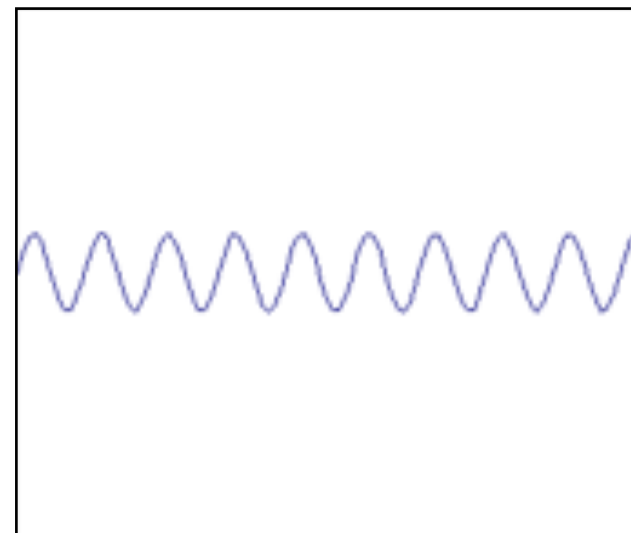




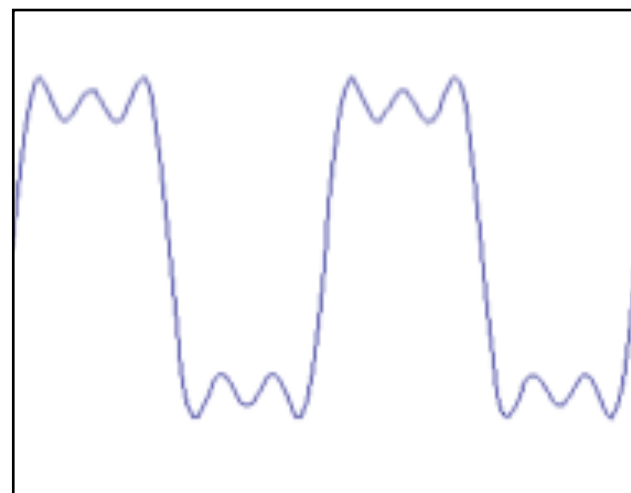
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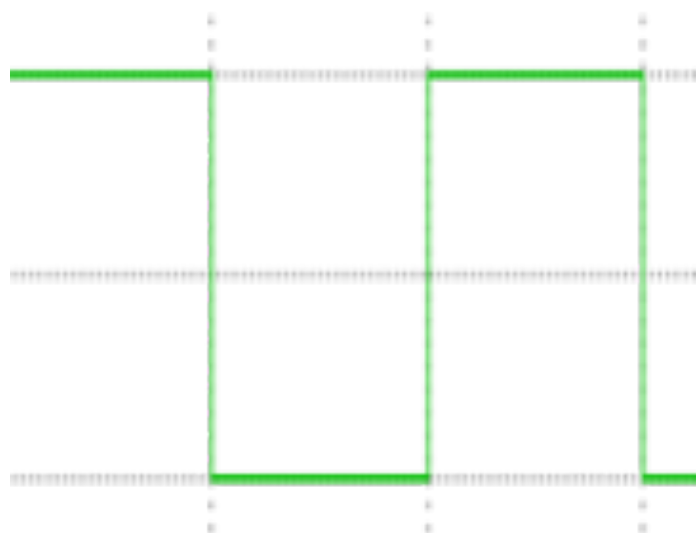


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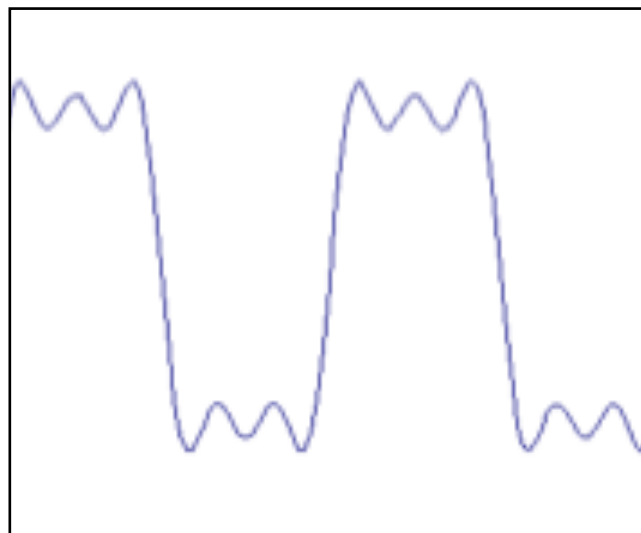


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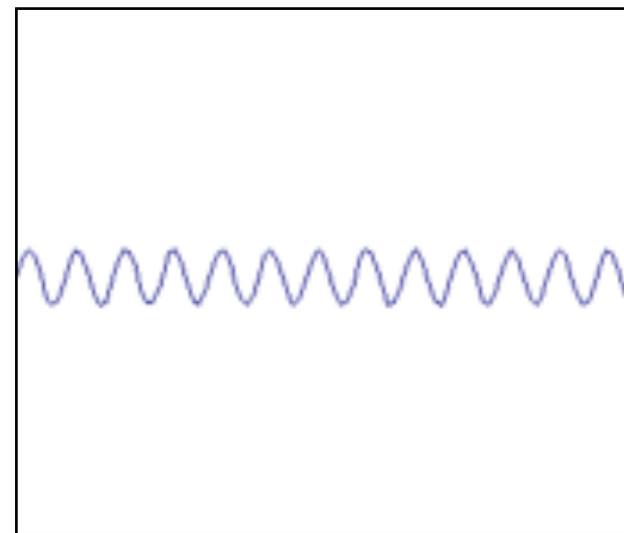




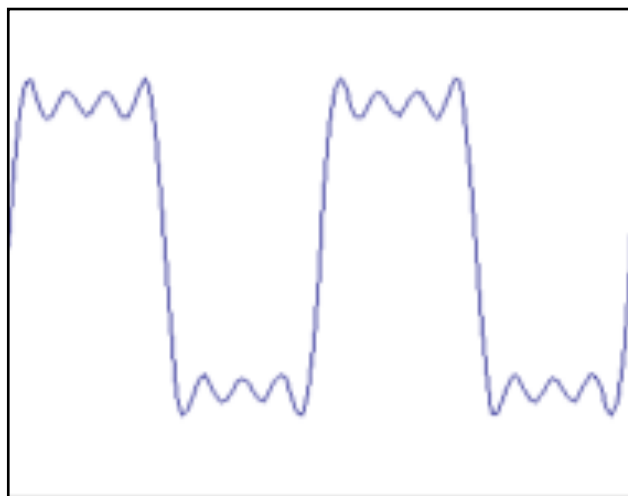
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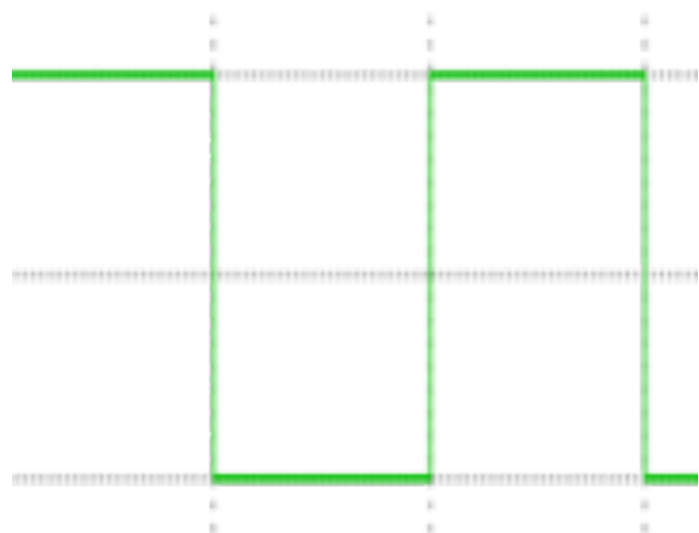


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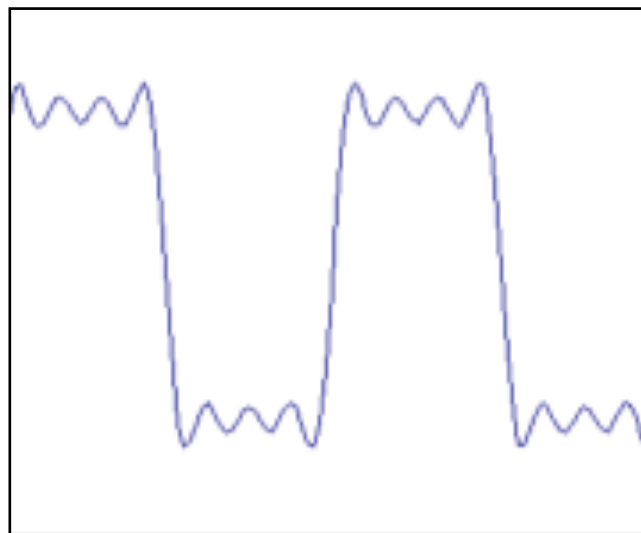


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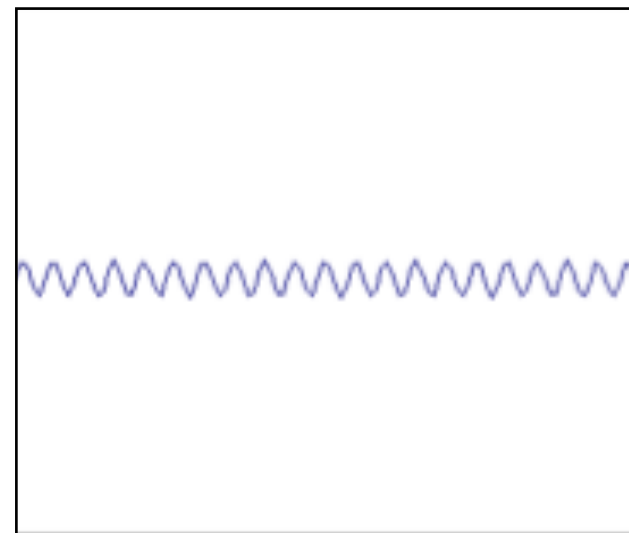




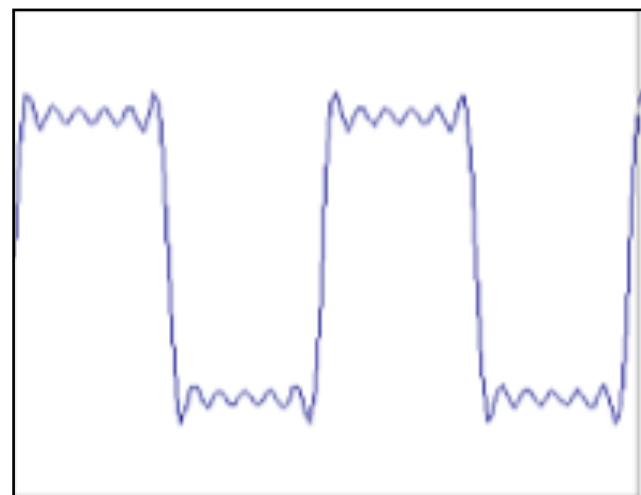
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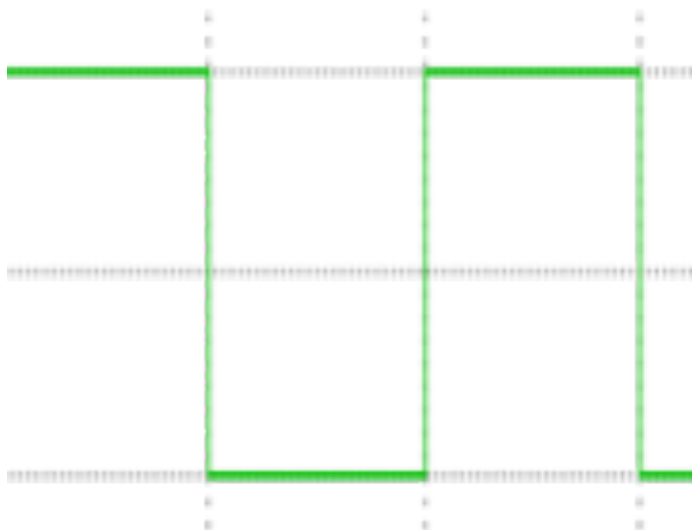


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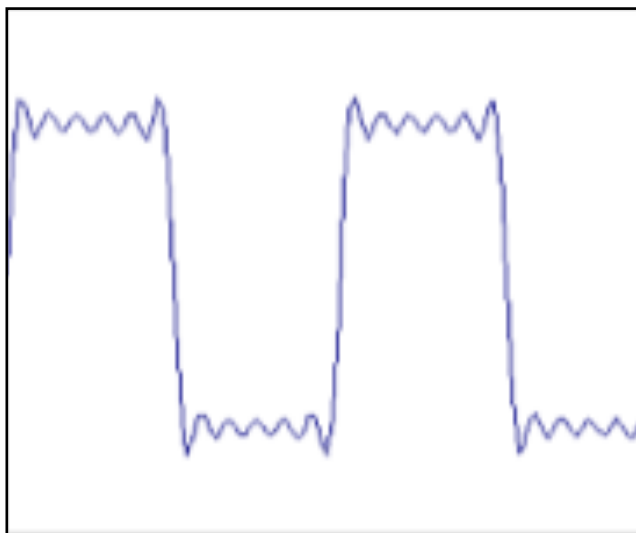


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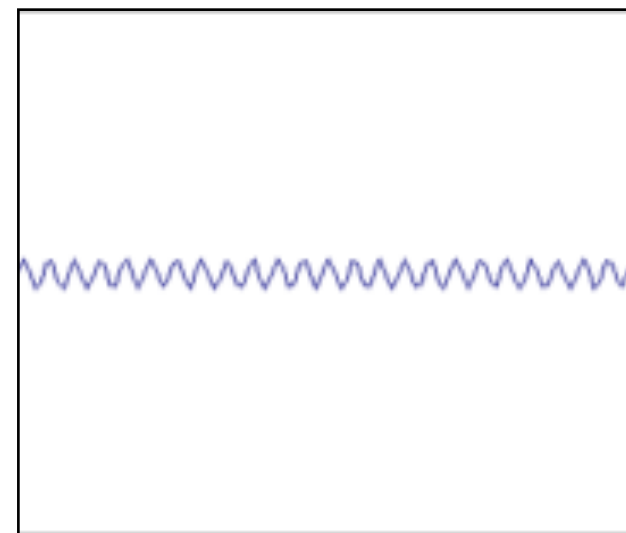




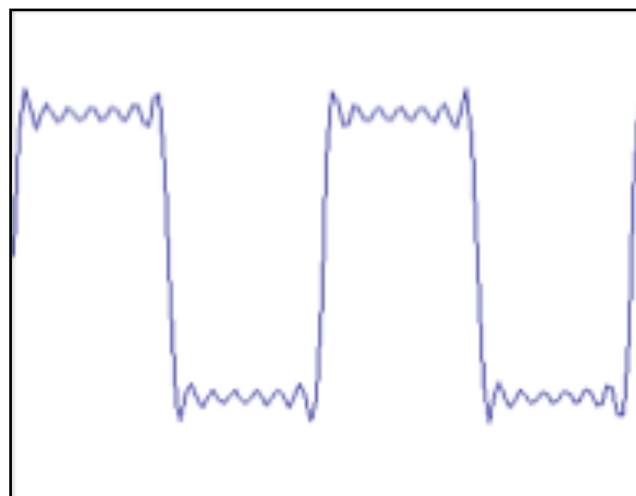
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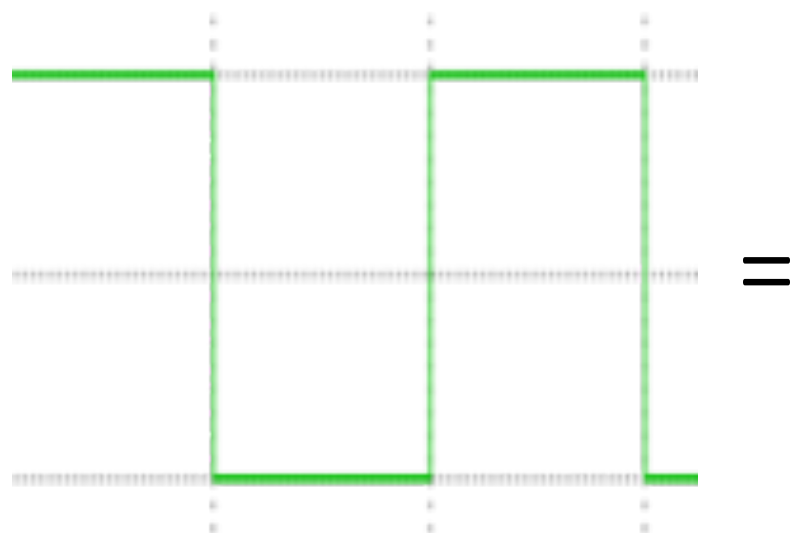
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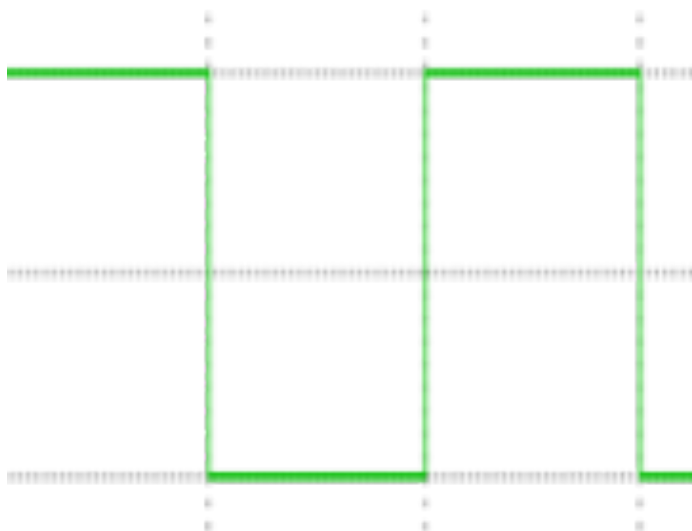
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Come lo possiamo
esprimere
matematicamente?



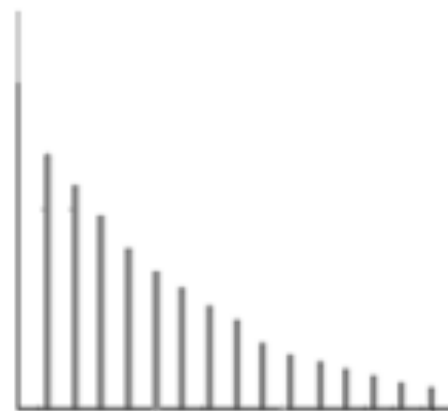
$$= A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kx)$$



=

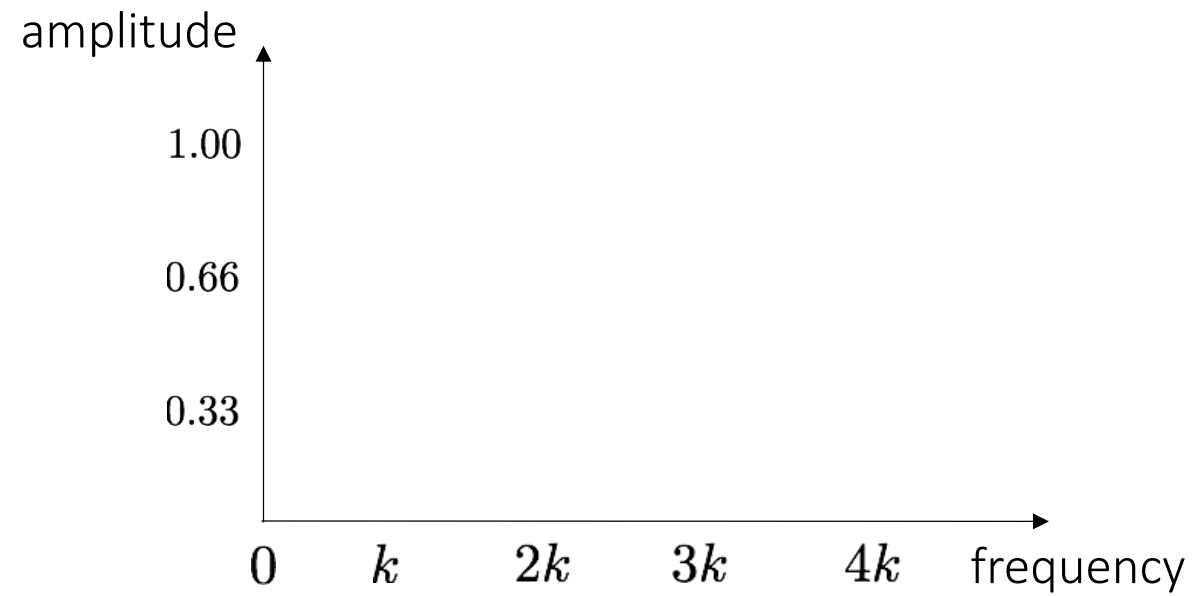
$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kx)$$

Ampiezza

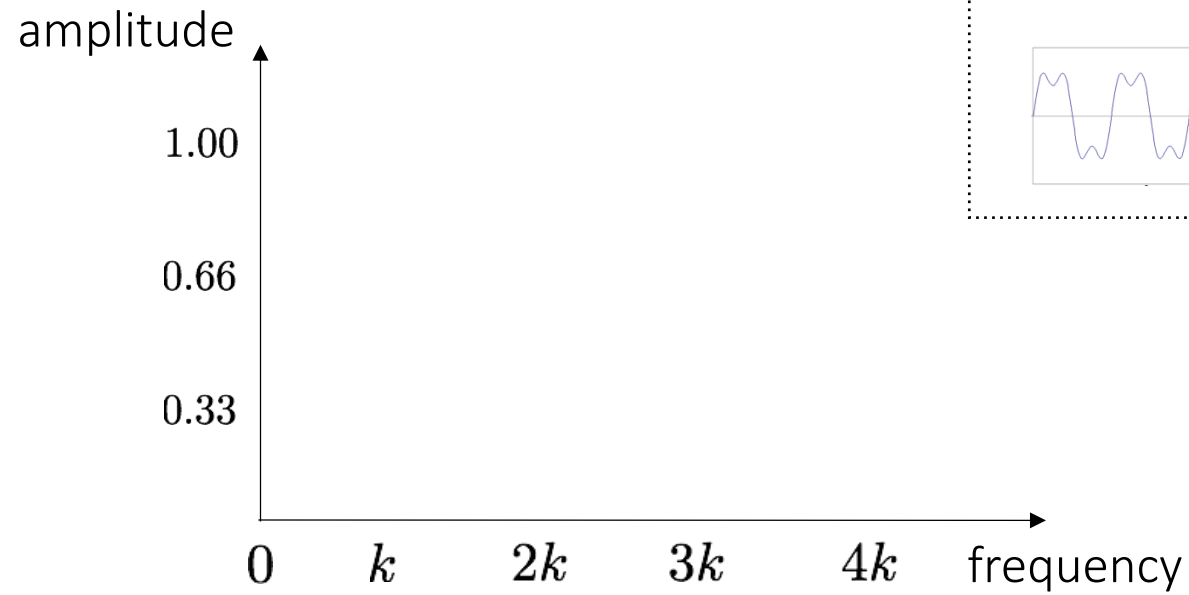


Frequenze

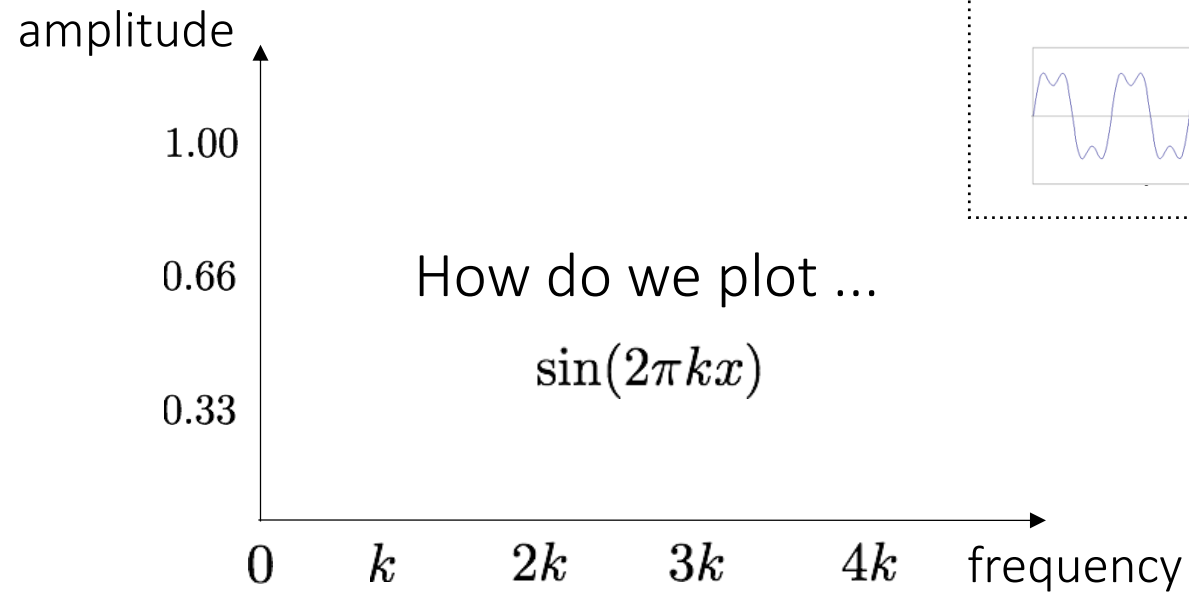
Lo spettro delle frequenze



Lo spettro delle frequenze



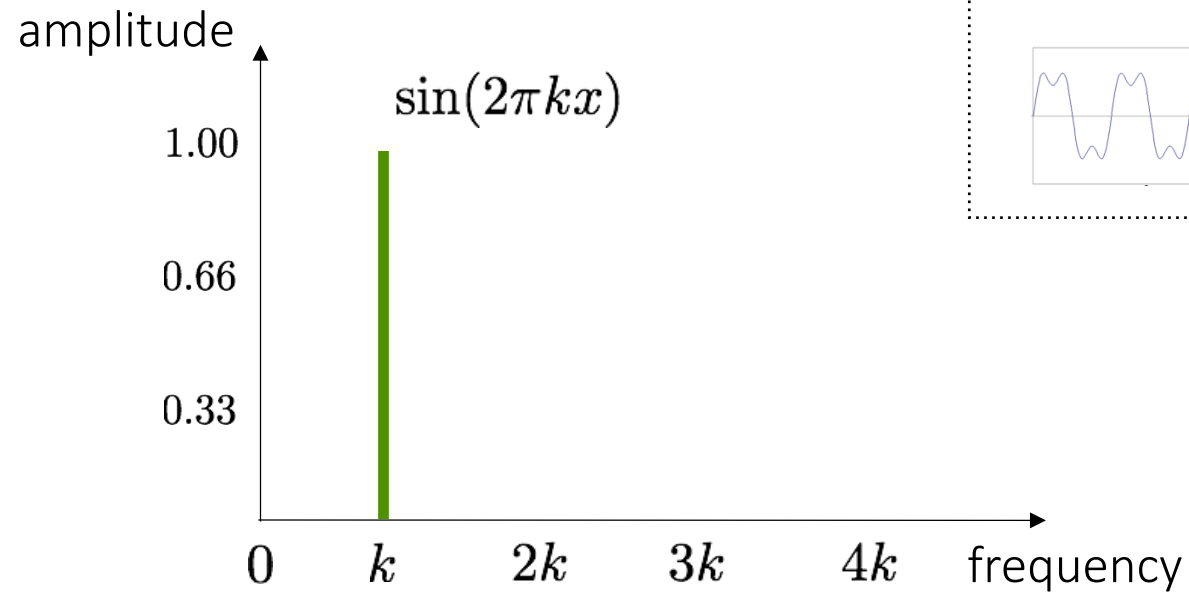
Lo spettro delle frequenze



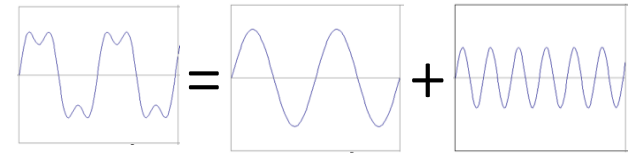
$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$



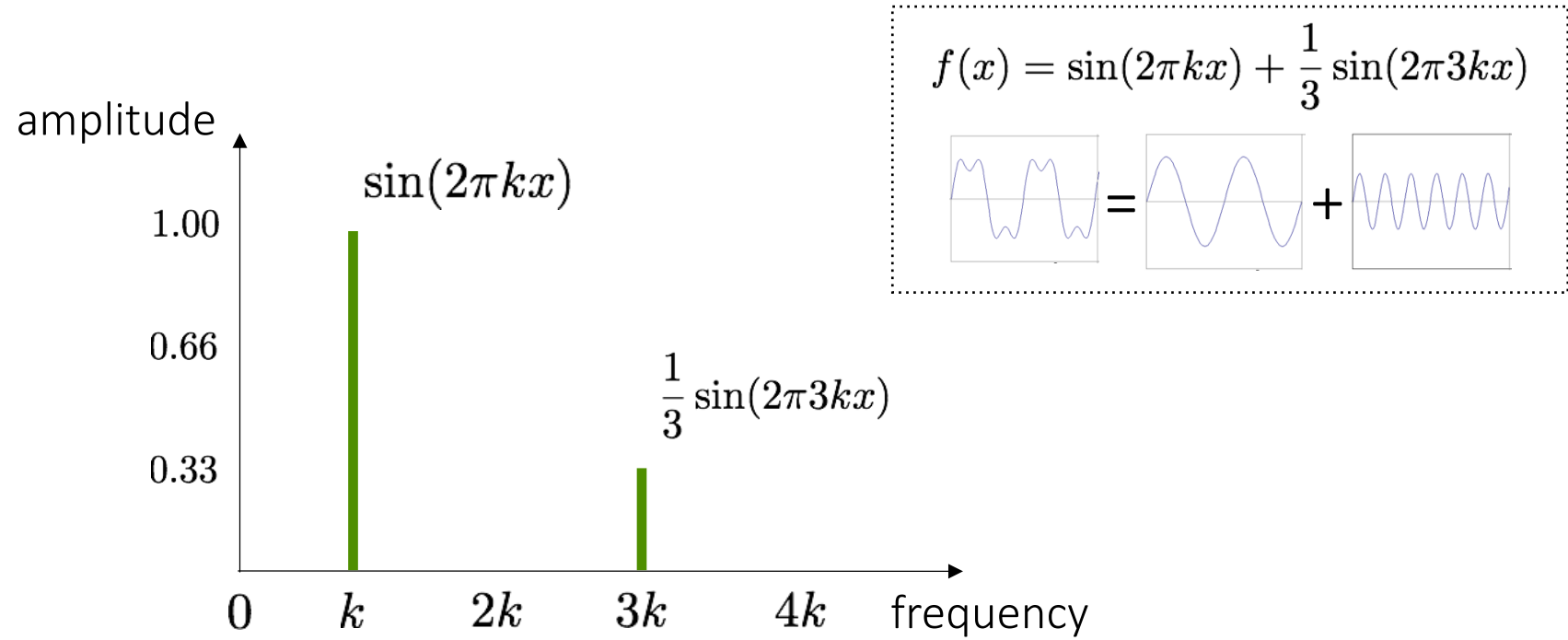
Lo spettro delle frequenze



$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$



Lo spettro delle frequenze



Da 1D a 2D

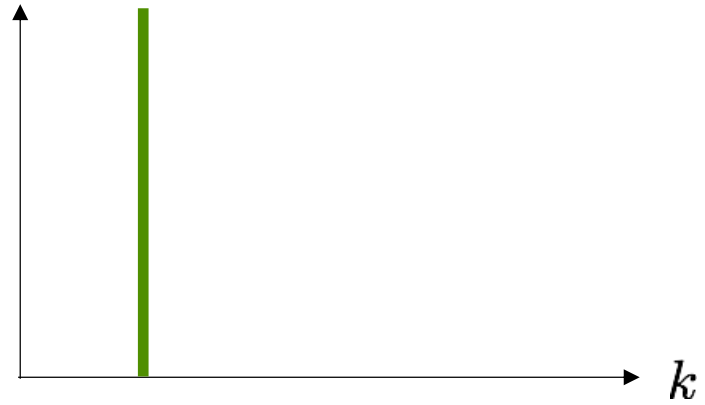
Dominio spaziale

Dominio frequenze

1D



$|F(k)|$



2D



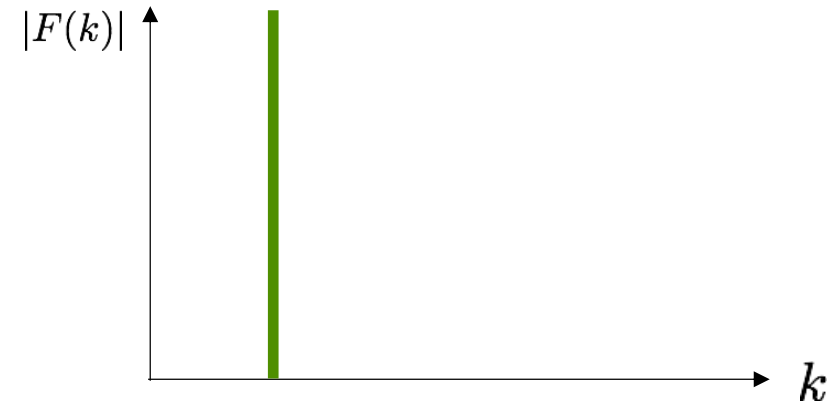
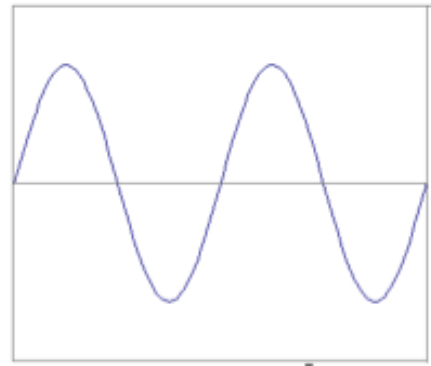
?

Da 1D a 2D

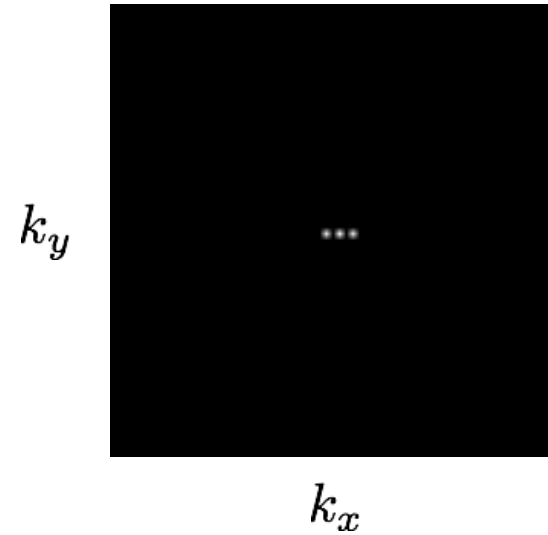
Dominio spaziale

Dominio frequenze

1D



2D



A cosa corrispondono i tre punti?

Da 1D a 2D

Dominio spaziale



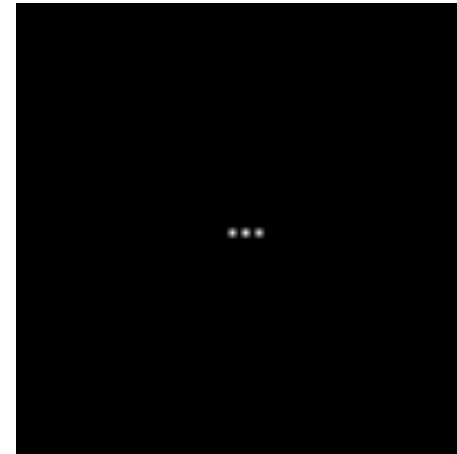
Dominio frequenze

?

k_y

...

k_x

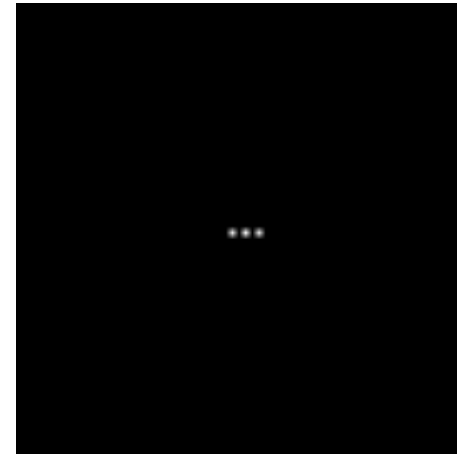
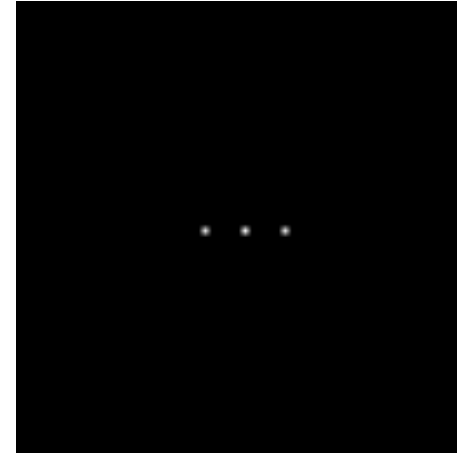


Da 1D a 2D

Dominio spaziale



Dominio frequenze

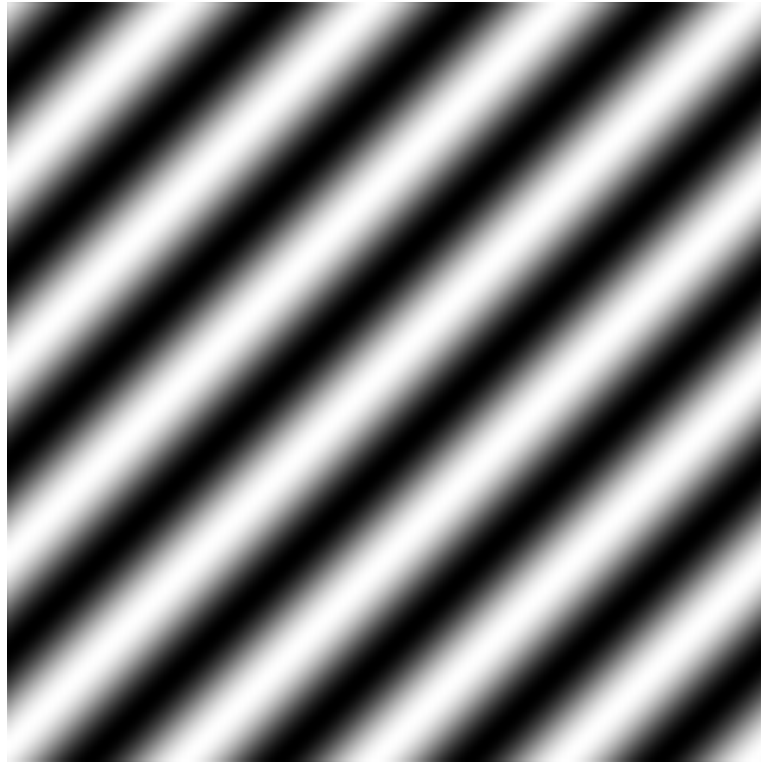


k_y

k_x

Esempio

Qual è il corrispondente di questa immagine?



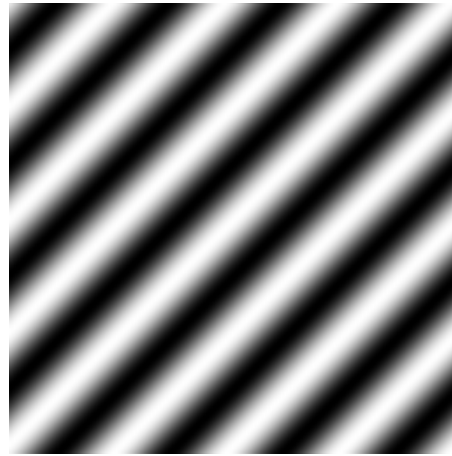
Esempio



Esempio



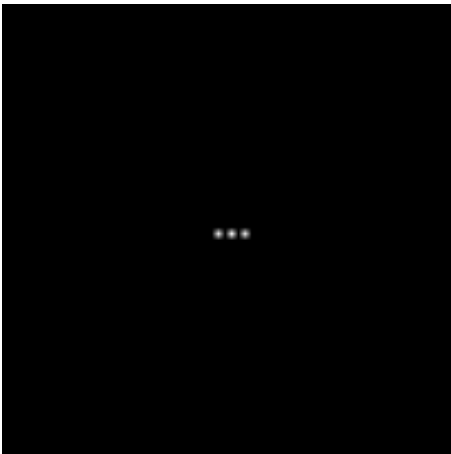
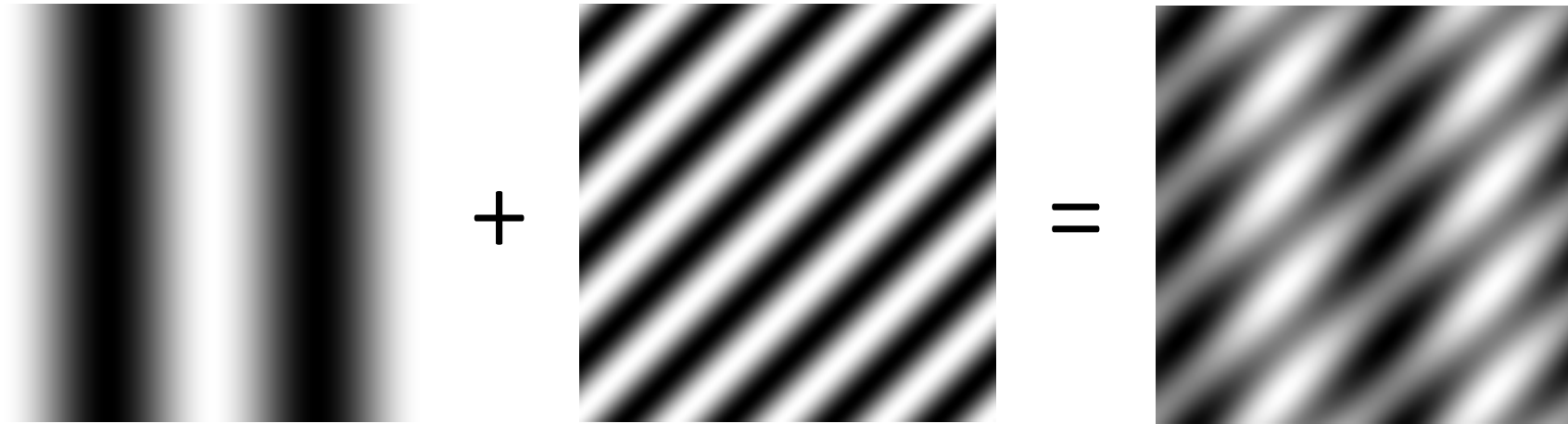
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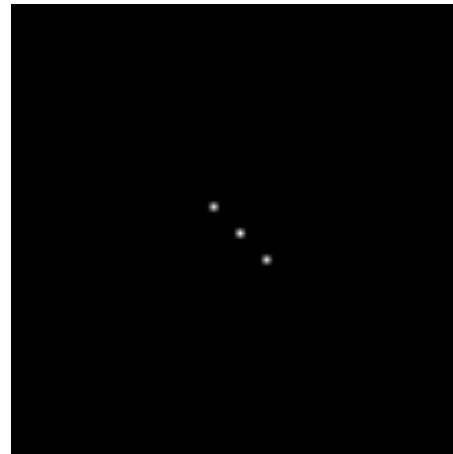
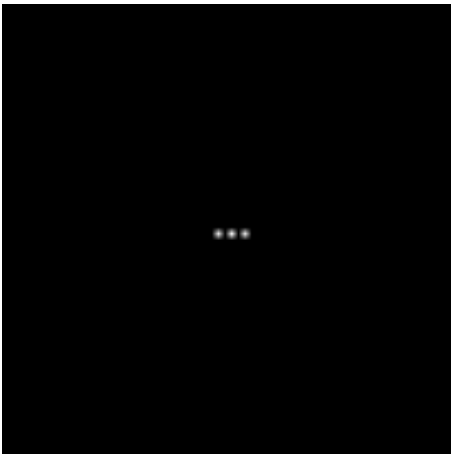
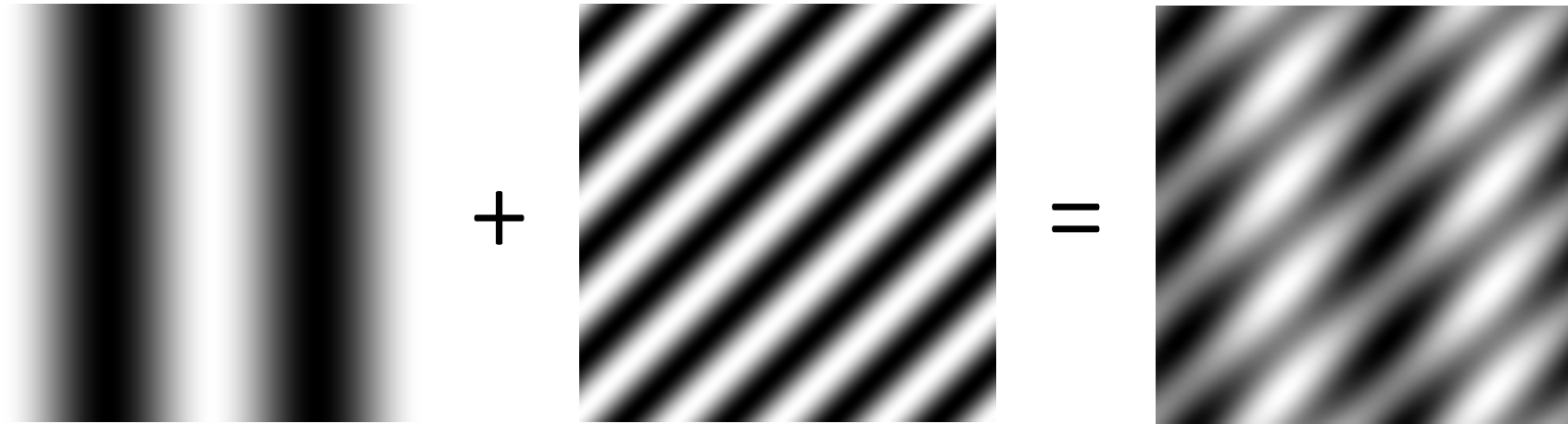
?

Esempio



?

Esempio



Trasformata di Fourier

Diretta

Inversa

Continuo

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi kx} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{j2\pi kx} dx$$

Discreto

$$F(k) = \sum_{k=0}^{N-1} f(x) e^{-j2\pi \frac{k}{N} x}$$

$k = 0, 1, 2, \dots, N-1$

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi \frac{k}{N} x}$$

$x = 0, 1, 2, \dots, N-1$

Trasformata di Fourier

Diretta

Inversa

1D

$$F(k) = \sum_{k=0}^{N-1} f(x) e^{-j2\pi \frac{k}{N} x}$$

$k = 0, 1, 2, \dots, N - 1$

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi \frac{k}{N} x}$$

$x = 0, 1, 2, \dots, N - 1$

2D

$$F(h, k) = \sum_{h=0}^{N-1} \sum_{k=0}^{M-1} f(x, y) e^{-j2\pi \left(\frac{xh}{N} + \frac{yk}{M} \right)}$$

$h = 0, 1, 2, \dots, N - 1, k = 0, 1, 2, \dots, M - 1$

$$f(x, y) = \frac{1}{NM} \sum_{h=0}^{N-1} \sum_{k=0}^{M-1} F(h, k) e^{j2\pi \left(\frac{xh}{N} + \frac{yk}{M} \right)}$$

$x = 0, 1, 2, \dots, N - 1, y = 0, 1, 2, \dots, M - 1$

Trasformata di Fourier nel dominio reale

$$\mathcal{F}(f(x, y)) = F(h, k) = |F(h, k)|e^{-j\phi(h, k)}$$

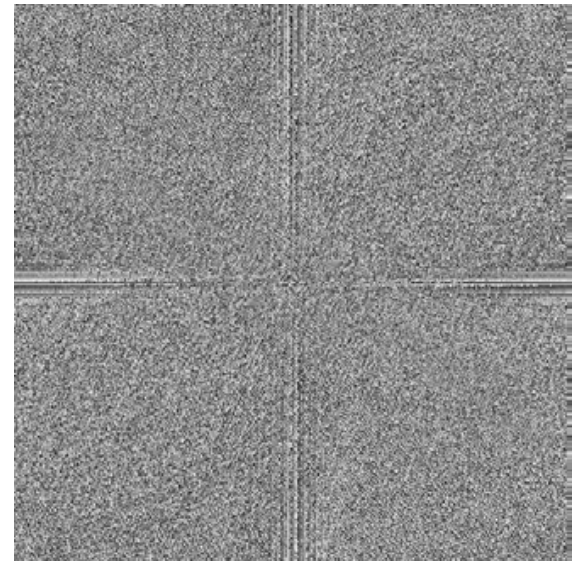
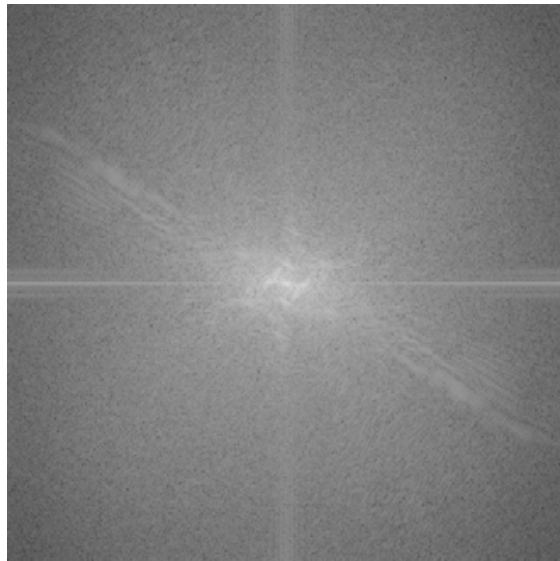
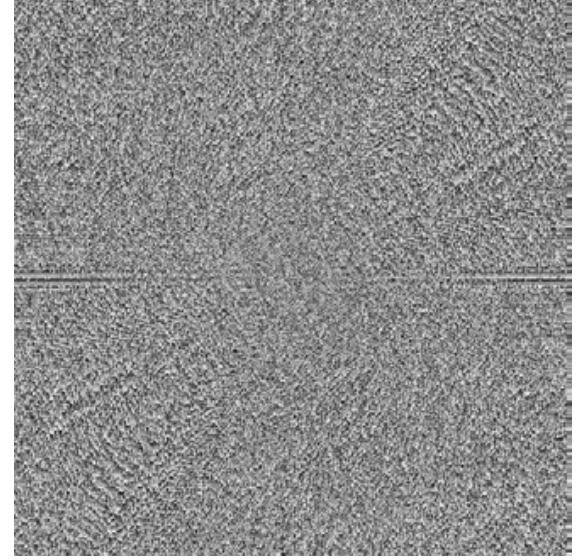
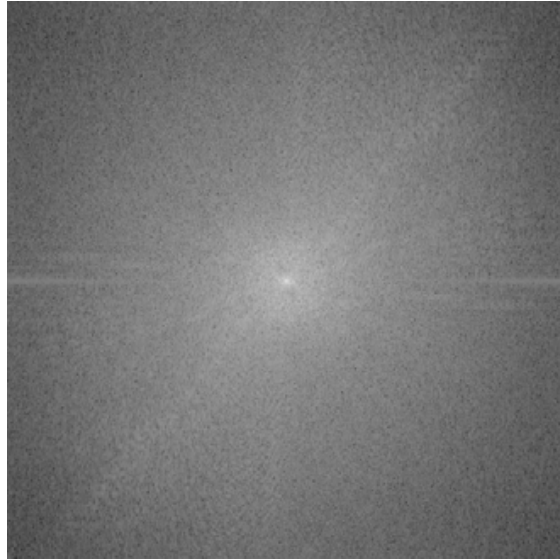
- Ampiezza

$$|F(h, k)| = \sqrt{R^2(h, k) + I^2(h, k)}$$

- Fase

$$\phi(h, k) = \tan^{-1} \frac{I(h, k)}{R(h, k)}$$

La trasformata di immagini



Immagine

Ampiezza

Fase

Applicazioni della FT

- Frequency-Domain Filtering

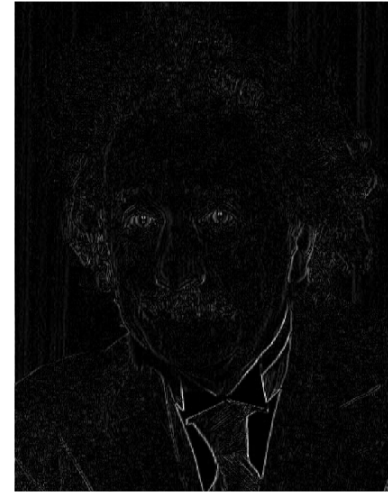
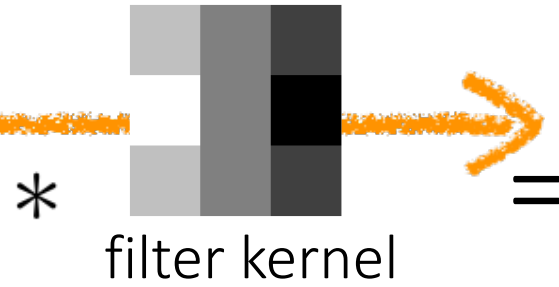
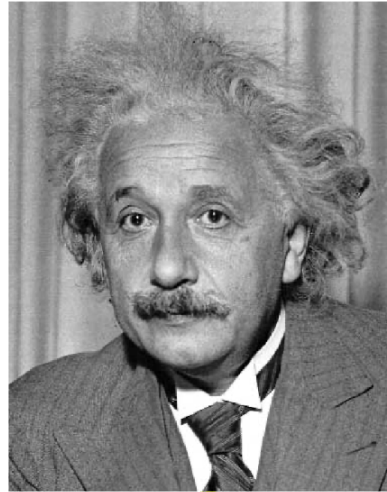
- Teorema di convoluzione

$$f(x, y) * h(x, y) = \mathcal{F}(f(x, y)) \cdot \mathcal{F}(h(x, y))$$

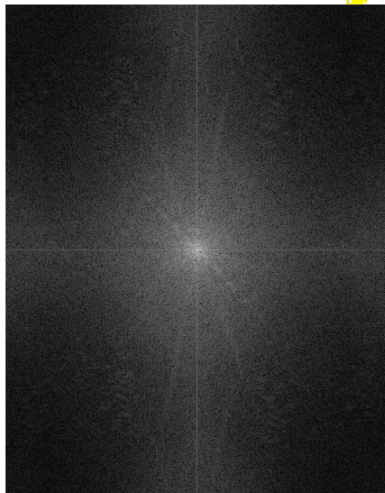
- Conseguenza

- Filtraggio come moltiplicazione di matrici
- Dominio spaziale -> FT->moltiplicazione ->IFT

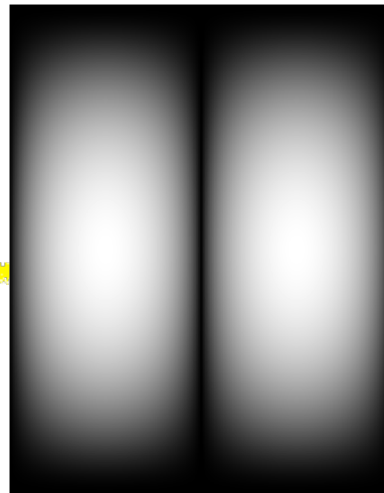
Esempio



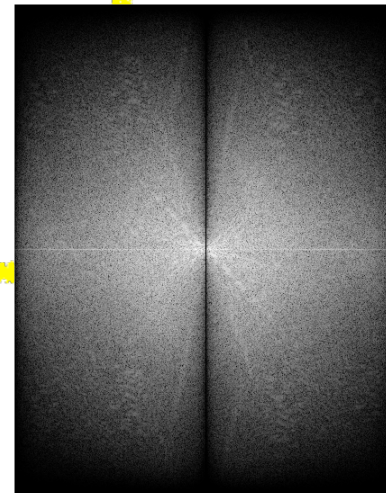
Fourier transform



\times



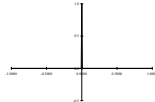
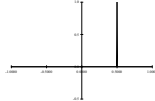



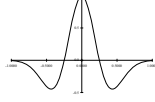
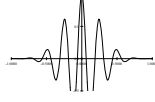

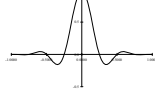
$=$



inverse Fourier transform



Alcune trasformate utili

Name	Signal	Transform
impulse	 $\delta(x)$	$\Leftrightarrow 1$
shifted impulse	 $\delta(x - u)$	$\Leftrightarrow e^{-j\omega u}$
box filter	 $\text{box}(x/a)$	$\Leftrightarrow \text{sinc}(a\omega)$
tent	 $\text{tent}(x/a)$	$\Leftrightarrow \text{sinc}^2(a\omega)$
Gaussian	 $G(x; \sigma)$	$\Leftrightarrow \frac{\sqrt{2\pi}}{\sigma} G(\omega; \sigma^{-1})$
Laplacian of Gaussian	 $(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2})G(x; \sigma)$	$\Leftrightarrow -\frac{\sqrt{2\pi}}{\sigma} \omega^2 G(\omega; \sigma^{-1})$
Gabor	 $\cos(\omega_0 x)G(x; \sigma)$	$\Leftrightarrow \frac{\sqrt{2\pi}}{\sigma} G(\omega \pm \omega_0; \sigma^{-1})$
unsharp mask	 $(1 + \gamma)\delta(x) - \gamma G(x; \sigma)$	$\Leftrightarrow (1 + \gamma) - \frac{\sqrt{2\pi}\gamma}{\sigma} G(\omega; \sigma^{-1})$
windowed sinc	 $\text{rcos}(x/(aW)) \text{sinc}(x/a)$	\Leftrightarrow (see Figure 3.29)