Analisi di Immagini e Video (Computer Vision)

Giuseppe Manco

Outline

- Image Processing avanzato
  - Edge detection
  - Fourier Transform

Crediti

- Slides adattate da vari corsi
  - Analisi di Immagini (F. Angiulli) – Unical
  - Intro to Computer Vision (J. Tompkin) – CS Brown Edu
  - Computer Vision (I. Gkioulekas), CS CMU Edu

Analisi di Fourier

Trasformata di Fourier

$$A \sin(\omega x + \phi)$$

Ogni segnale periodico è una combinazione di queste componenti

Trasformata di Fourier

$$A\sin(\omega x + \phi)$$

Ampiezza

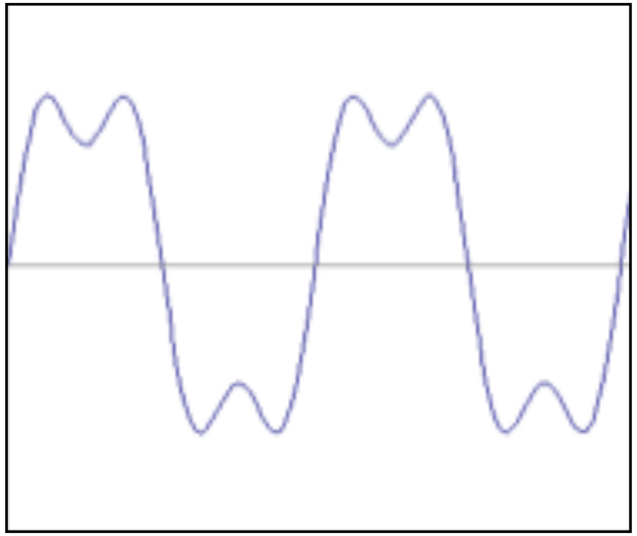
Sinusoide

Frequenza angolare

Variabile

Fase

Esempio



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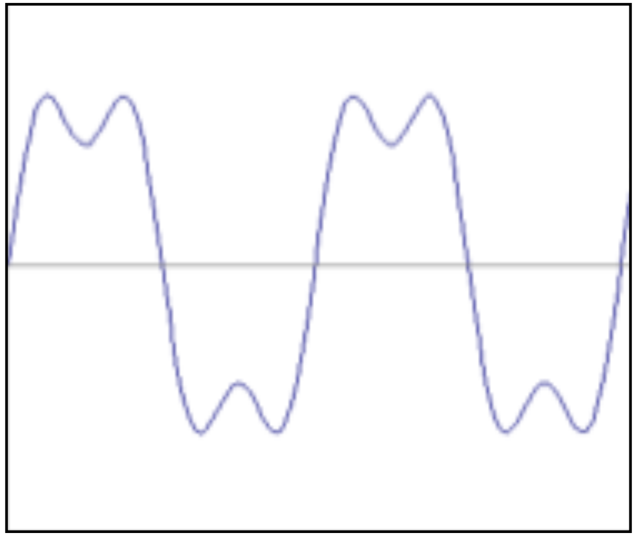

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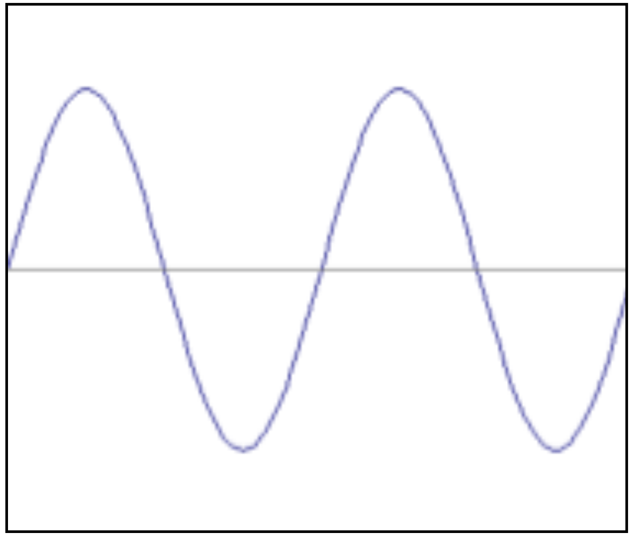
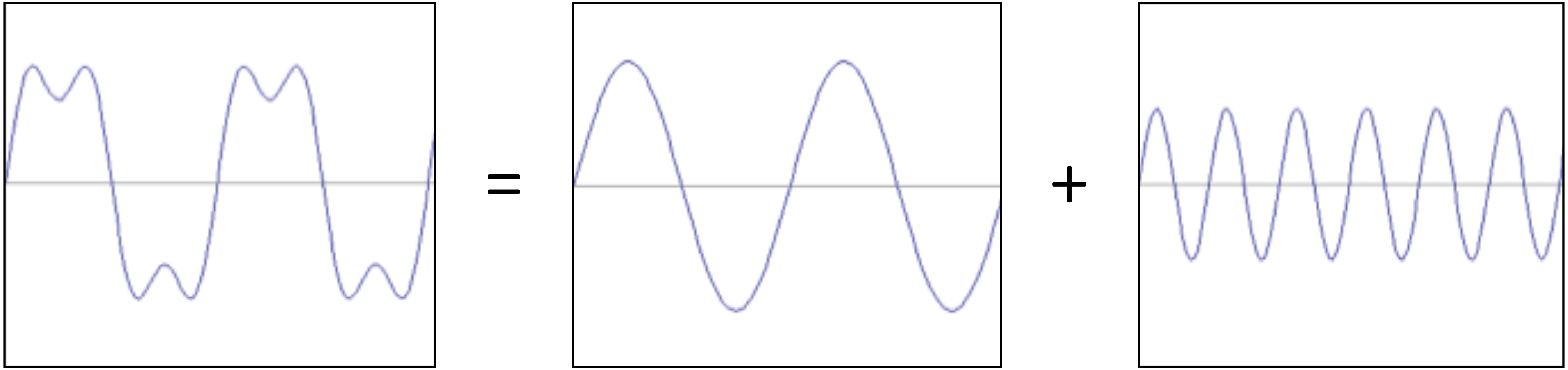


$\sin(2\pi x)$

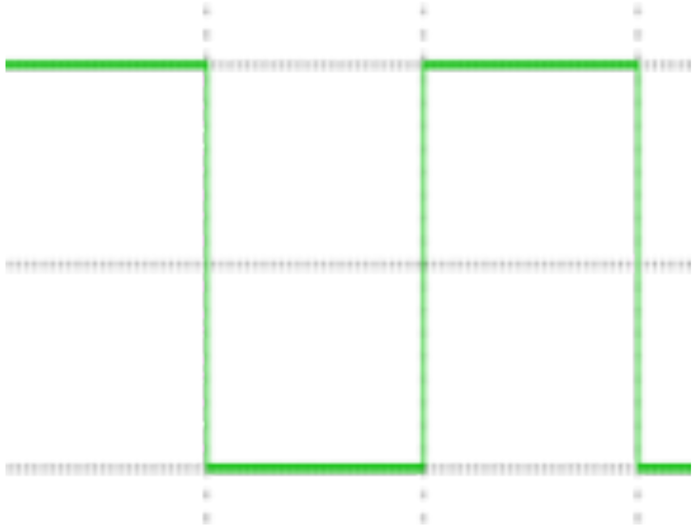
\+

?



$$f(x) = \sin(2\pi x) + \frac{1}{3}\sin(2\pi 3x) = \sin(2\pi x) + \frac{1}{3}\sin(2\pi 3x)$$

Esempio



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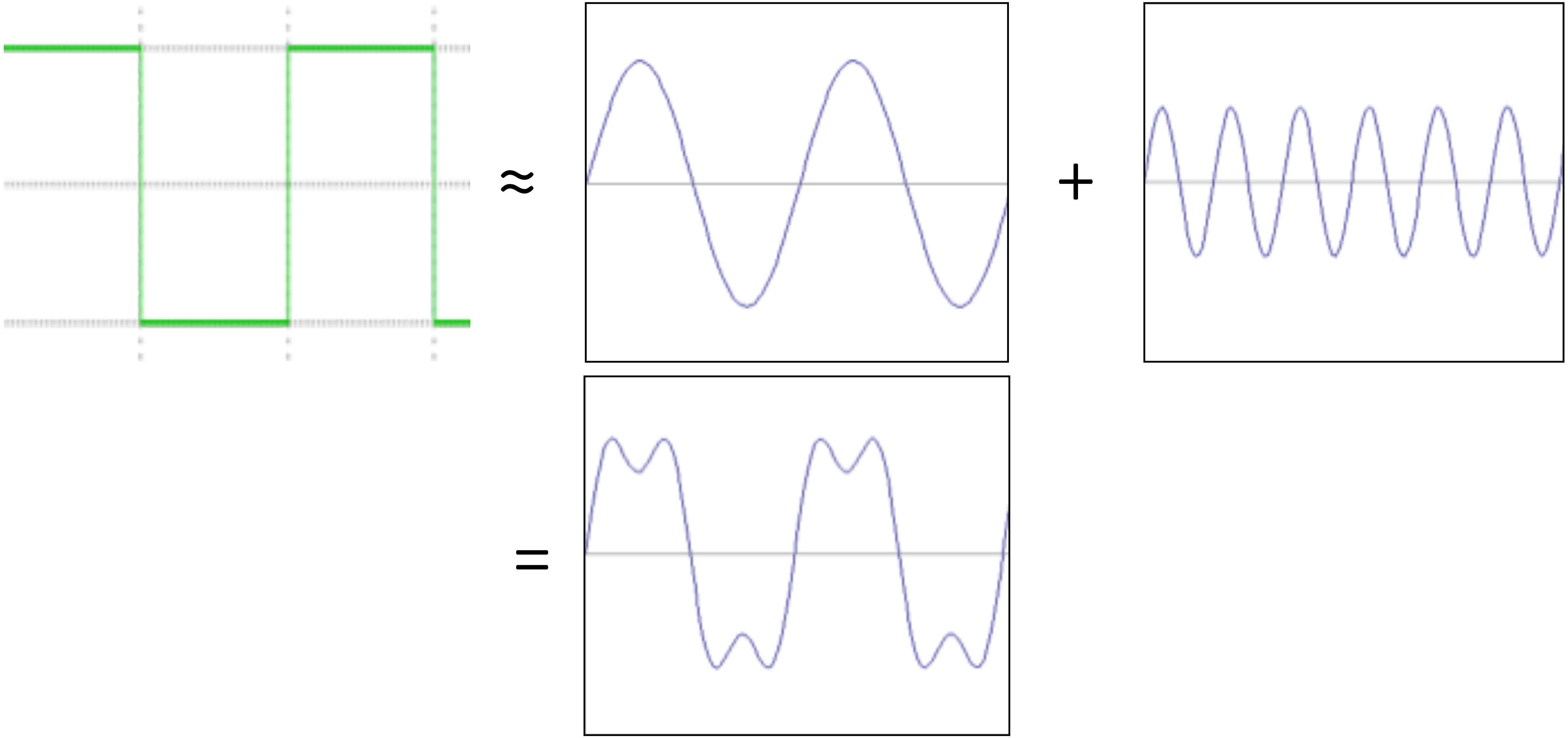
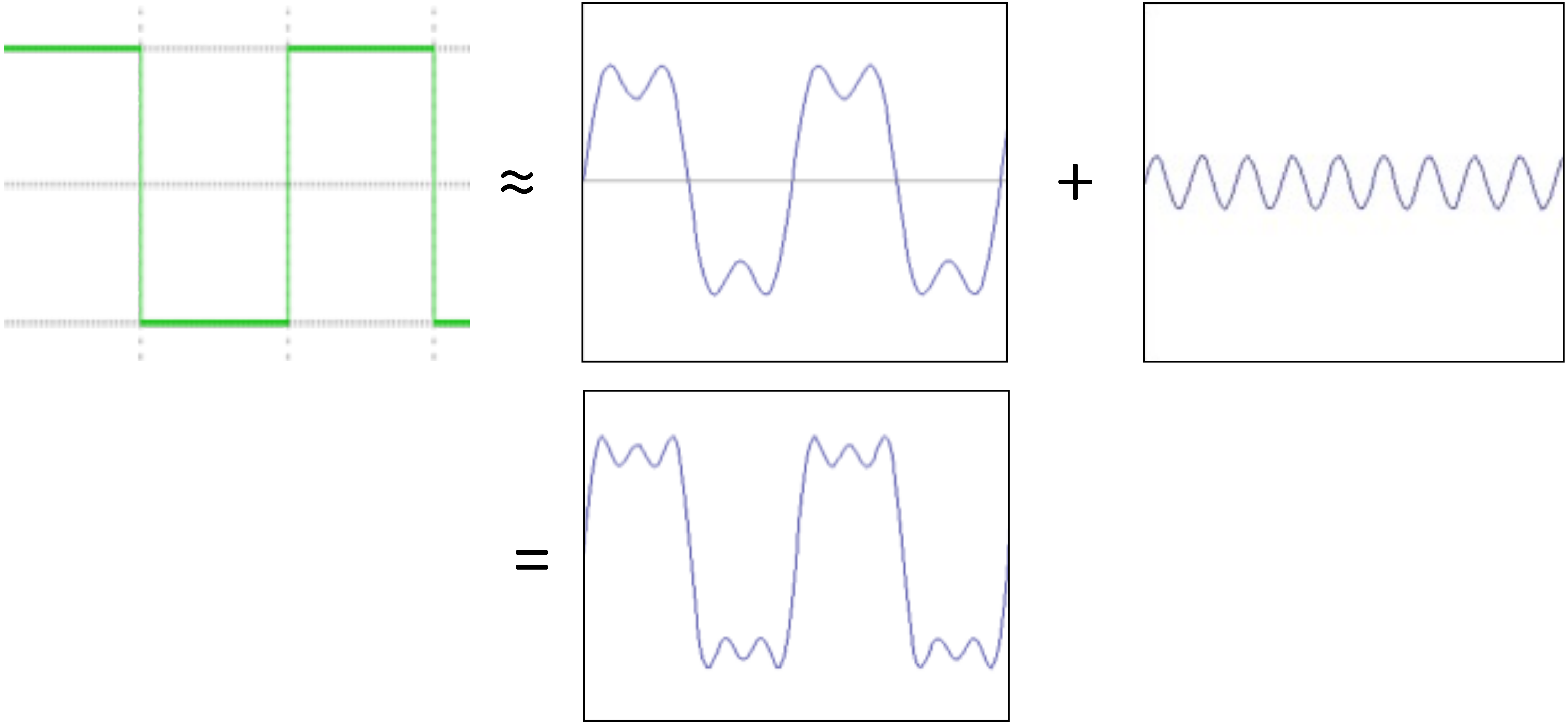
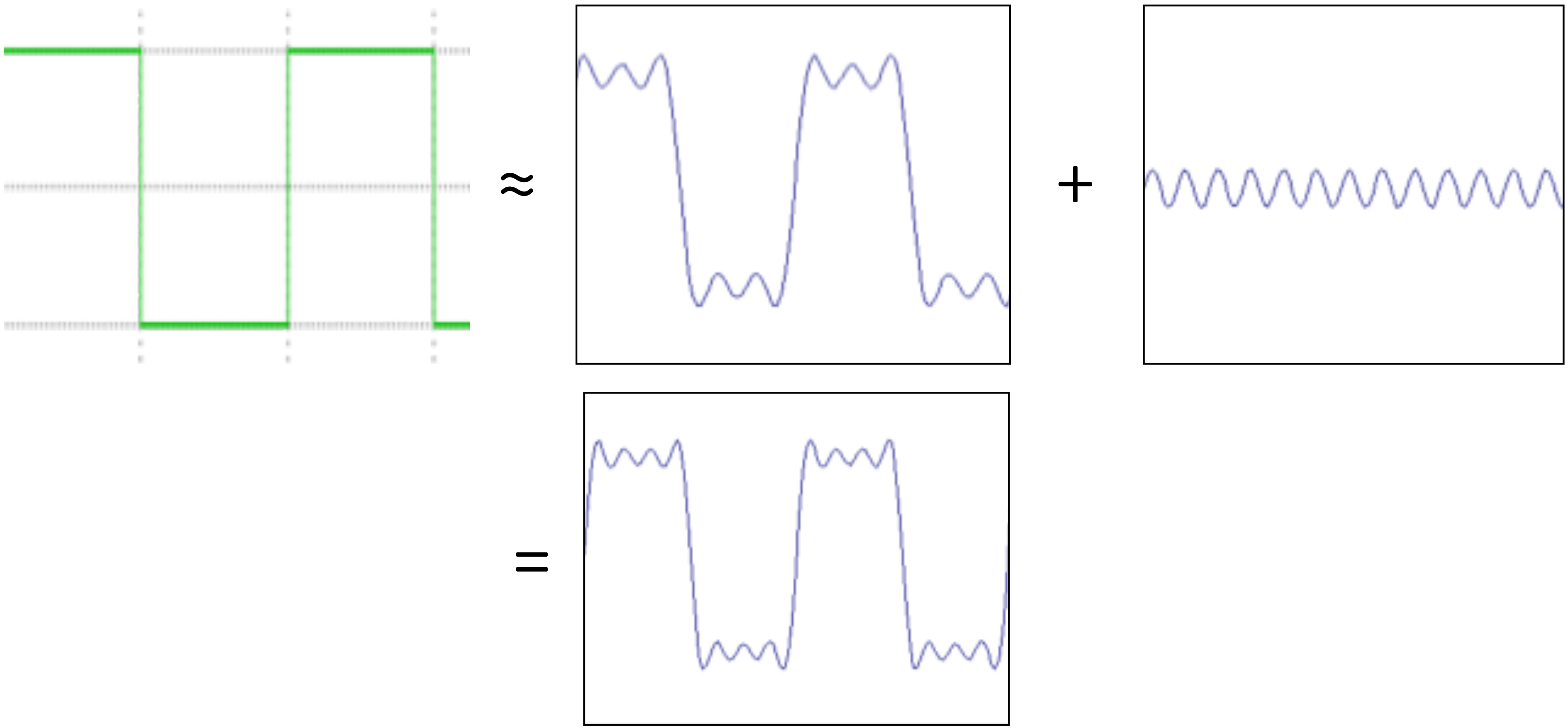
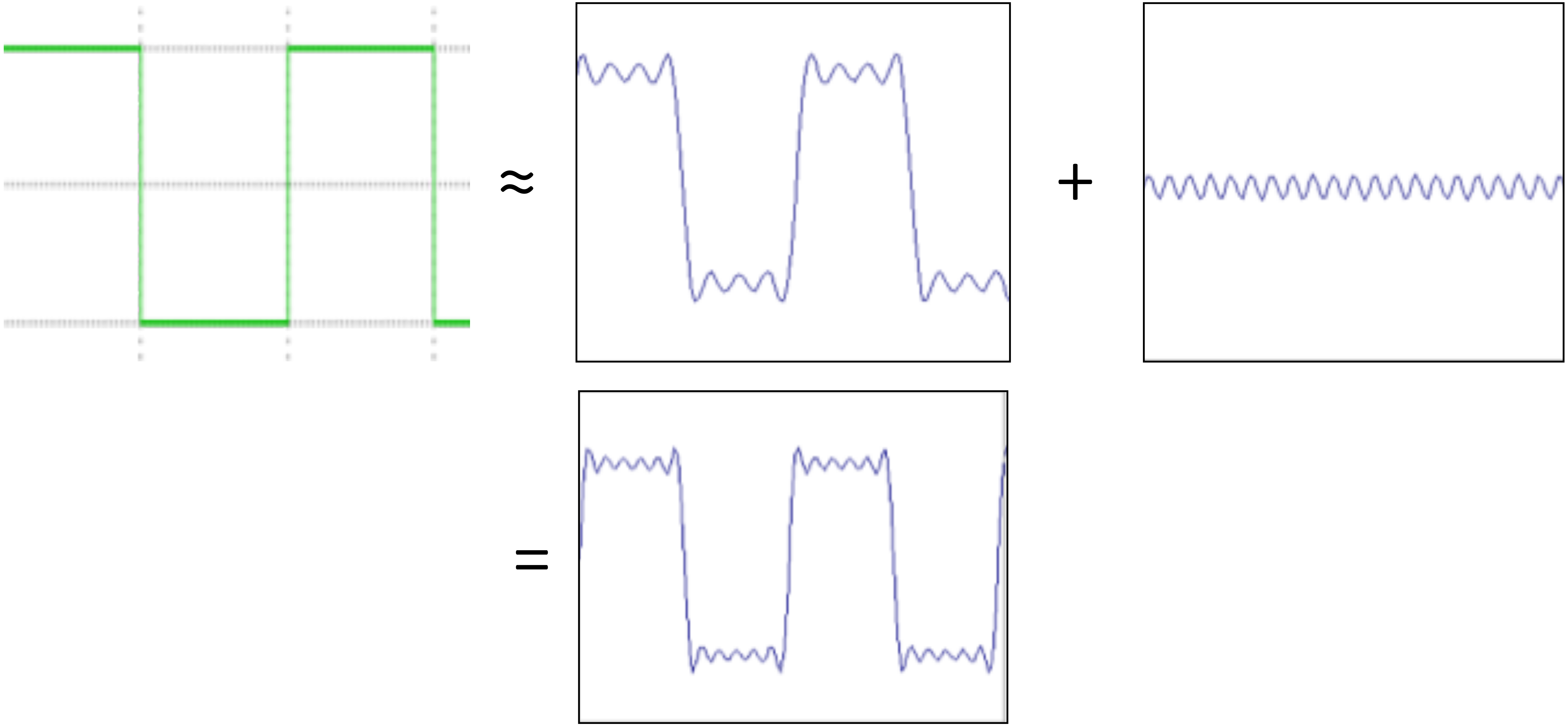
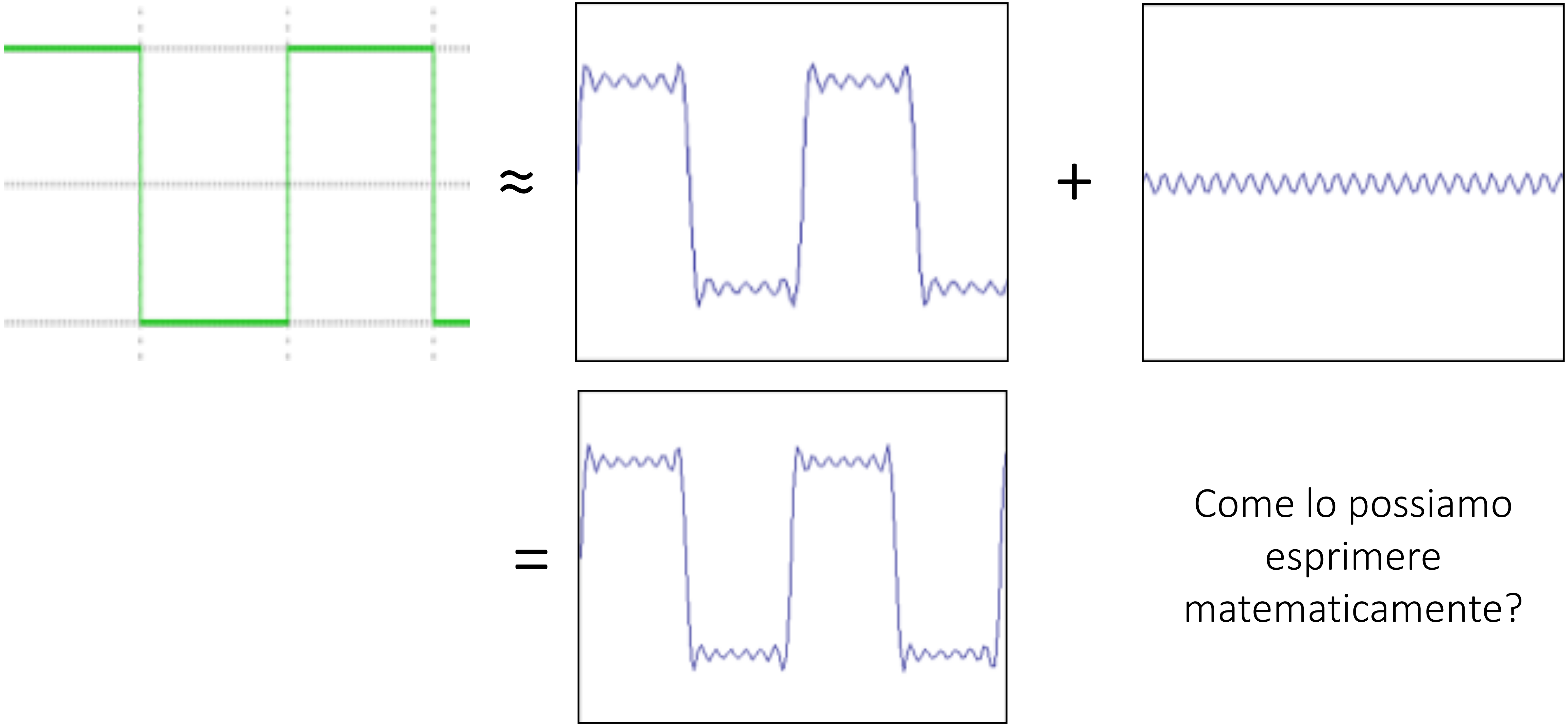


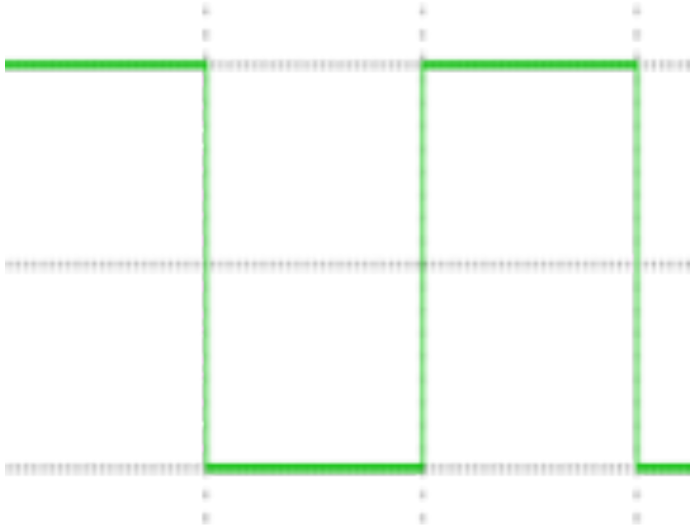




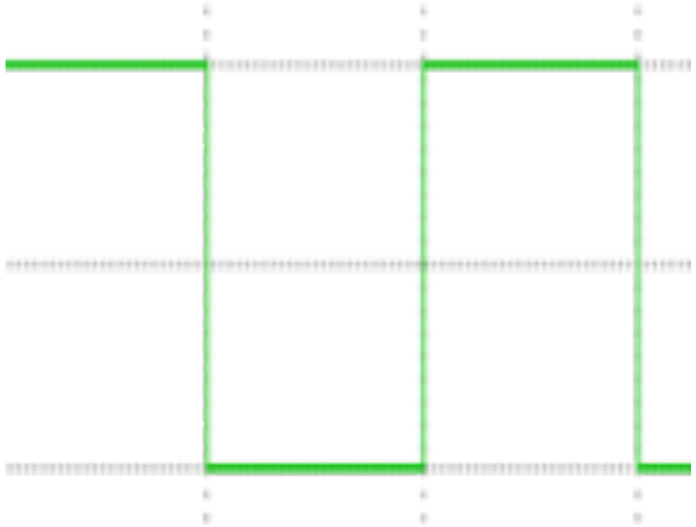




Come lo possiamo esprimere matematicamente?



$$= \quad A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kx)$$



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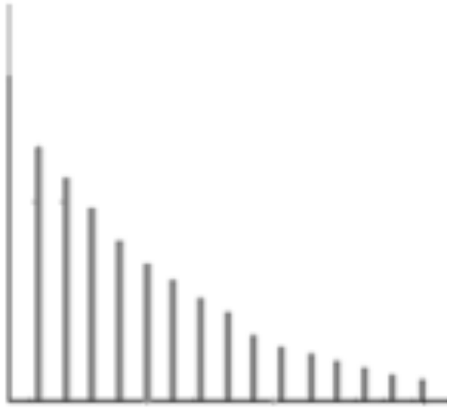
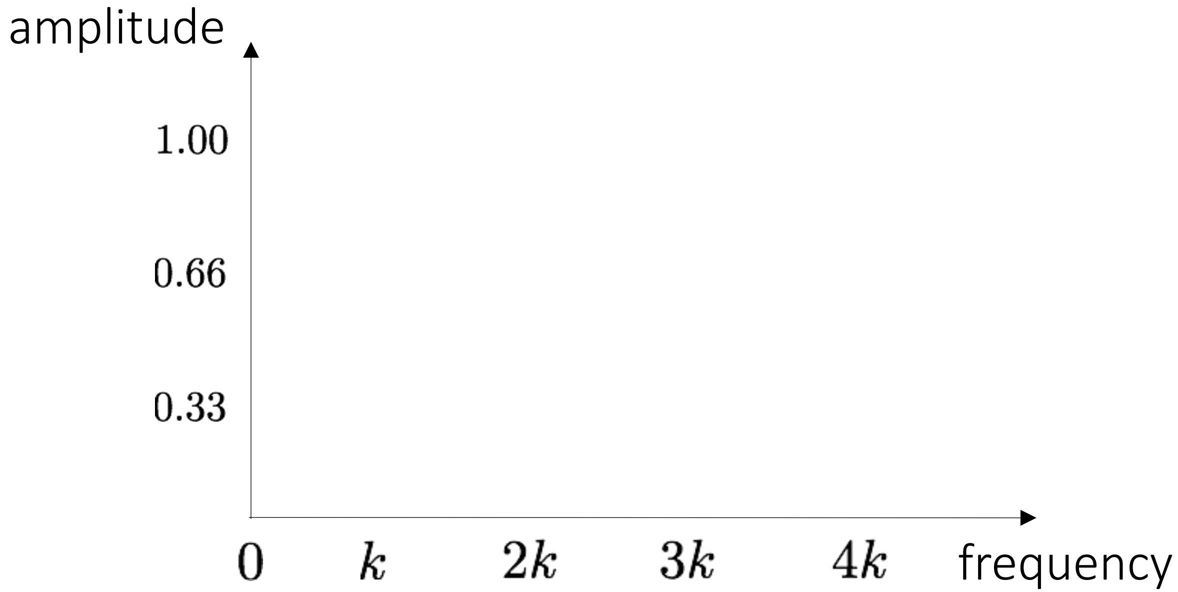
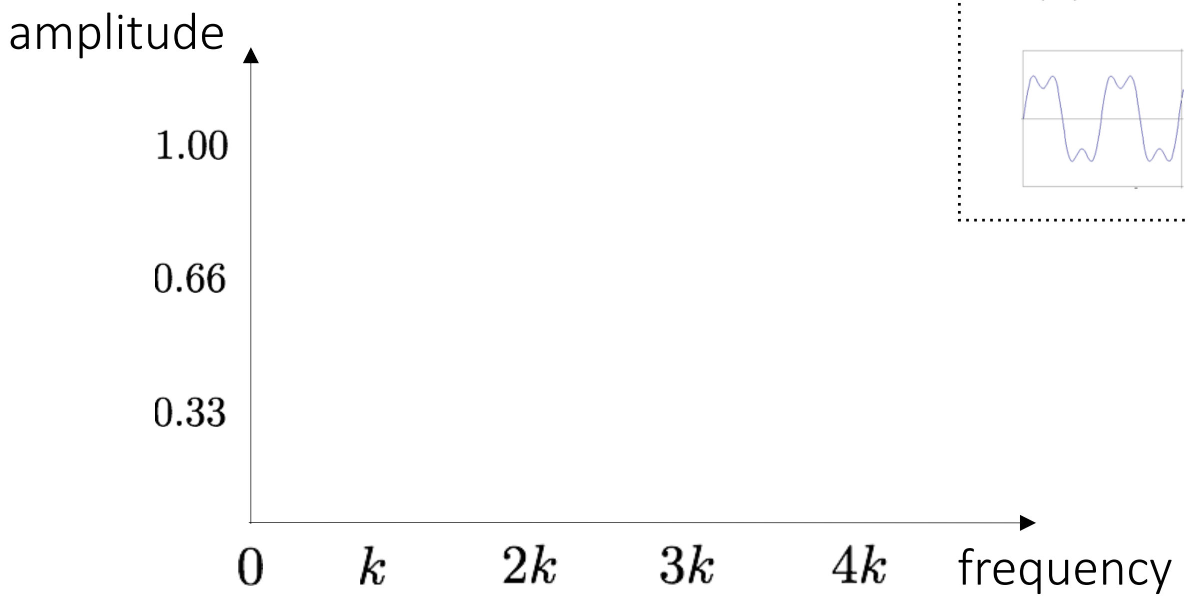
$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi k x)$$



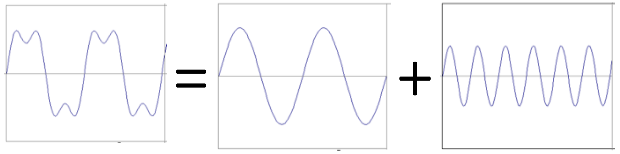
Lo spettro delle frequenze



Lo spettro delle frequenze

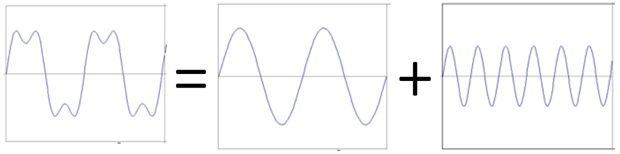


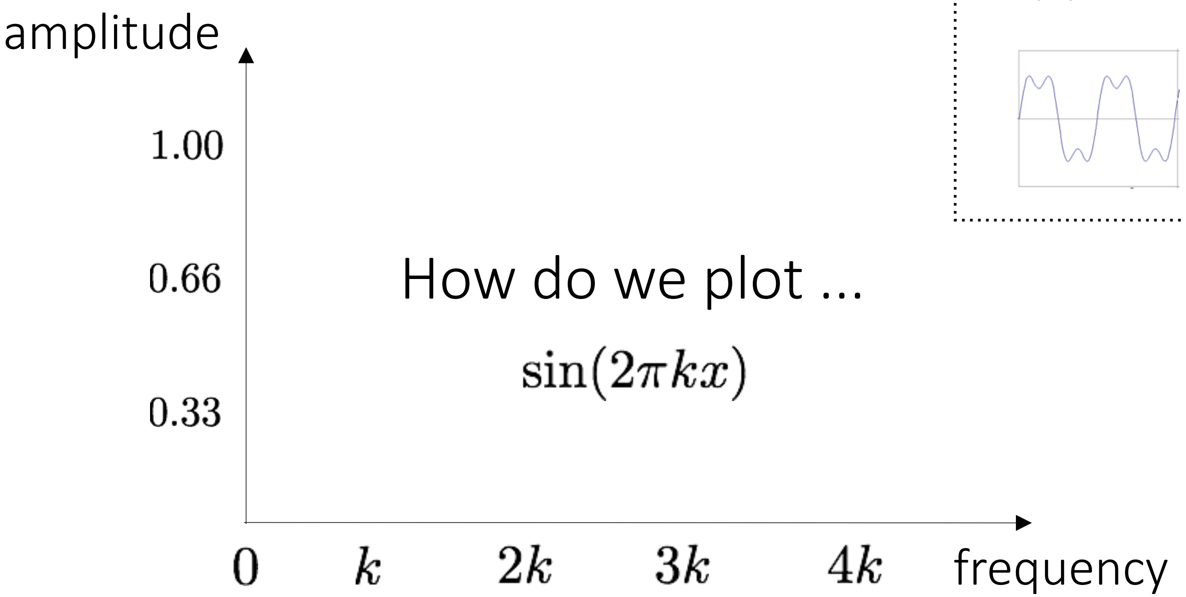
$$f(x) = \sin(2\pi k x) + \frac{1}{3}\sin(2\pi 3 k x)$$



Lo spettro delle frequenze

$$f(x) = \sin(2\pi kx) + \frac{1}{3}\sin(2\pi 3kx)$$

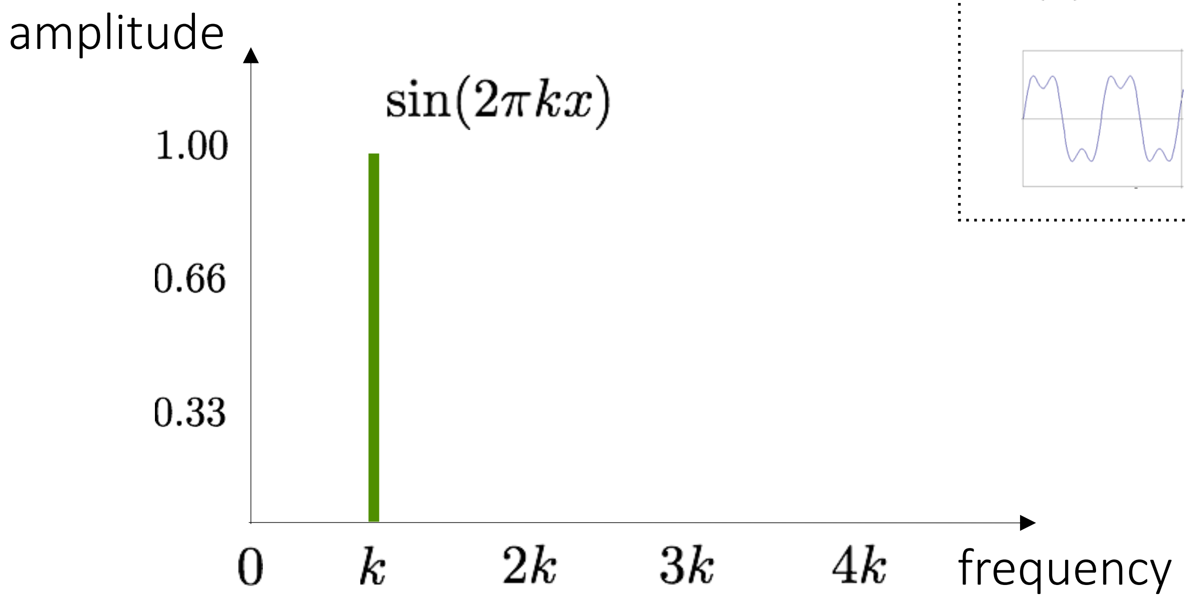




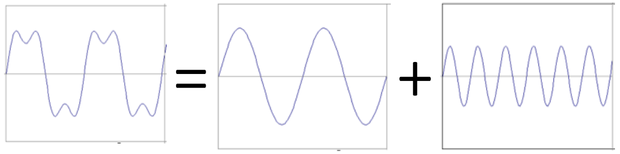
How do we plot ...

$$\sin(2\pi kx)$$

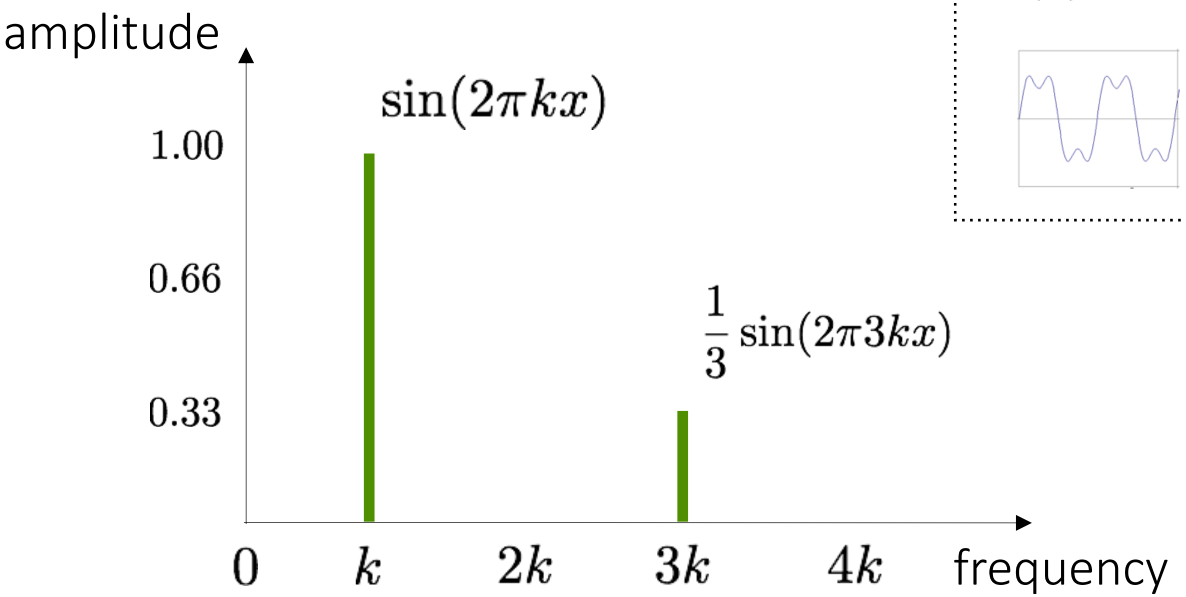
Lo spettro delle frequenze



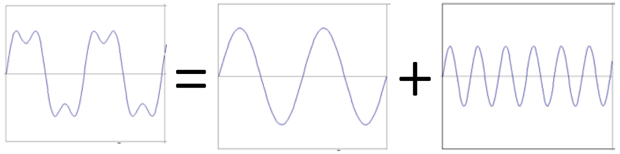
$$f(x) = \sin(2\pi kx) + \frac{1}{3}\sin(2\pi 3kx)$$



Lo spettro delle frequenze



$$f(x) = \sin(2\pi kx) + \frac{1}{3}\sin(2\pi 3kx)$$

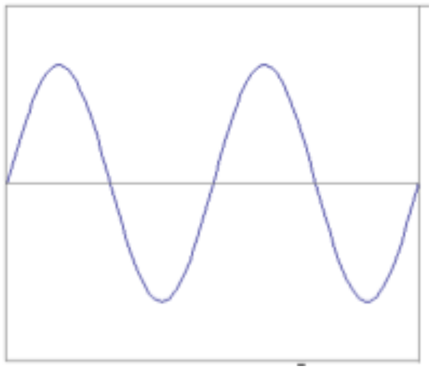


Da 1D a 2D

Dominio spaziale

Dominio frequenze

1D



2D

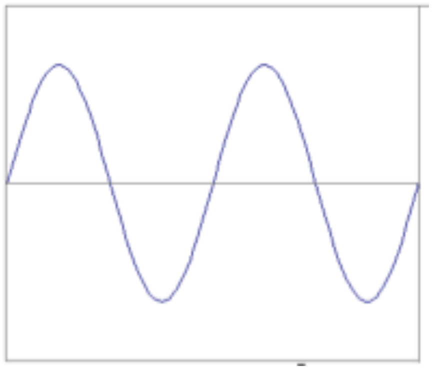
?

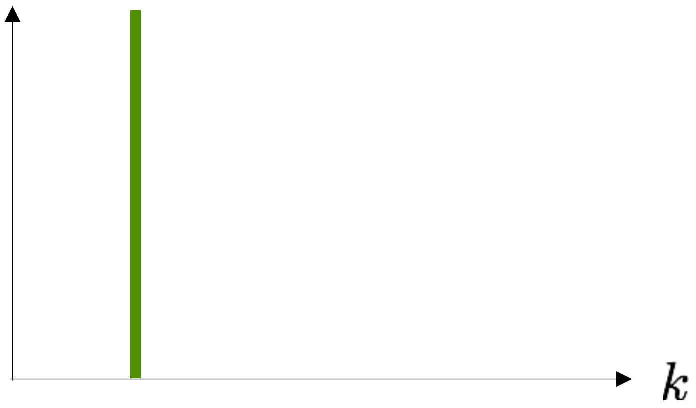
Da 1D a 2D

Dominio spaziale

Dominio frequenze

1D



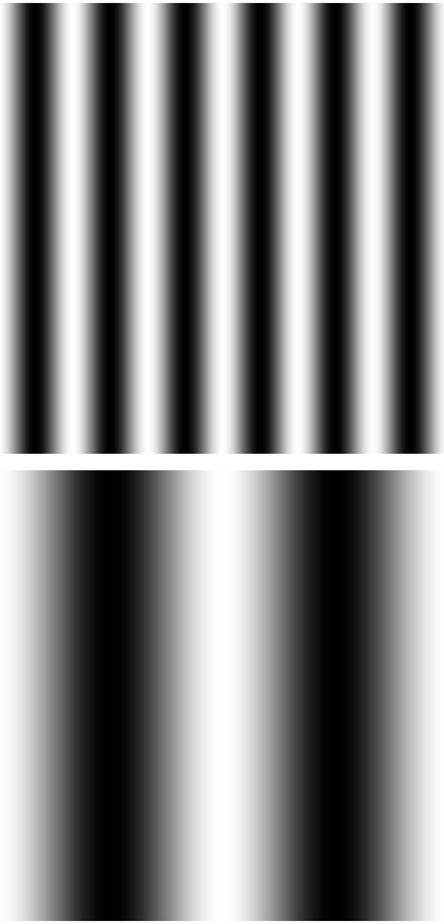


2D

A cosa corrispondono i tre punti?

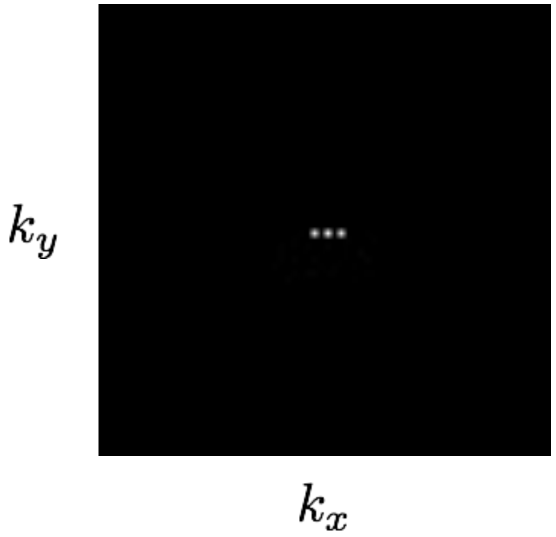
Da 1D a 2D

Dominio spaziale



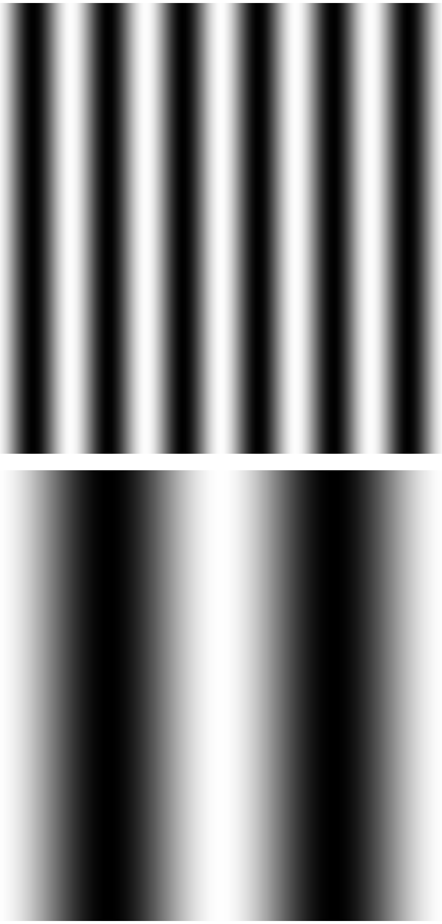
Dominio frequenze

?

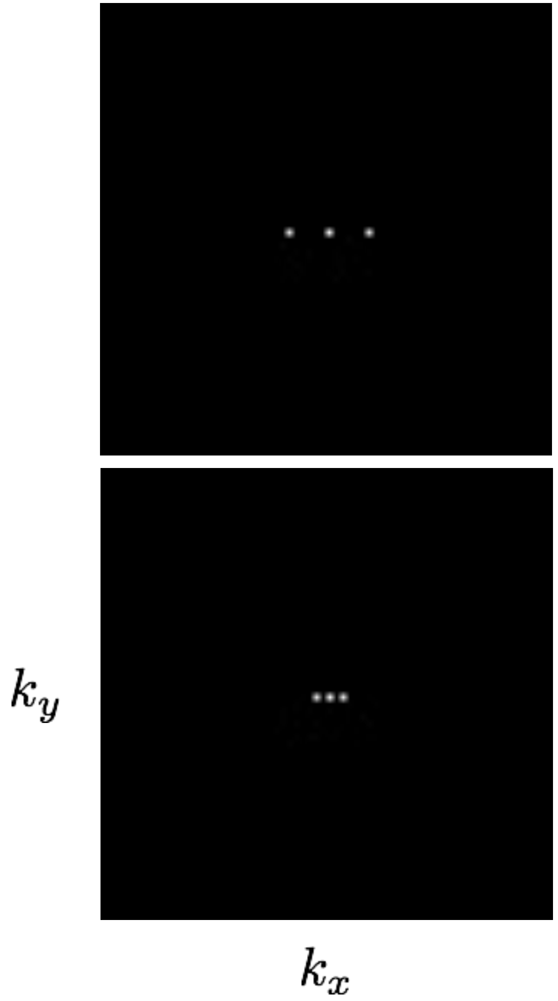


Da 1D a 2D

Dominio spaziale

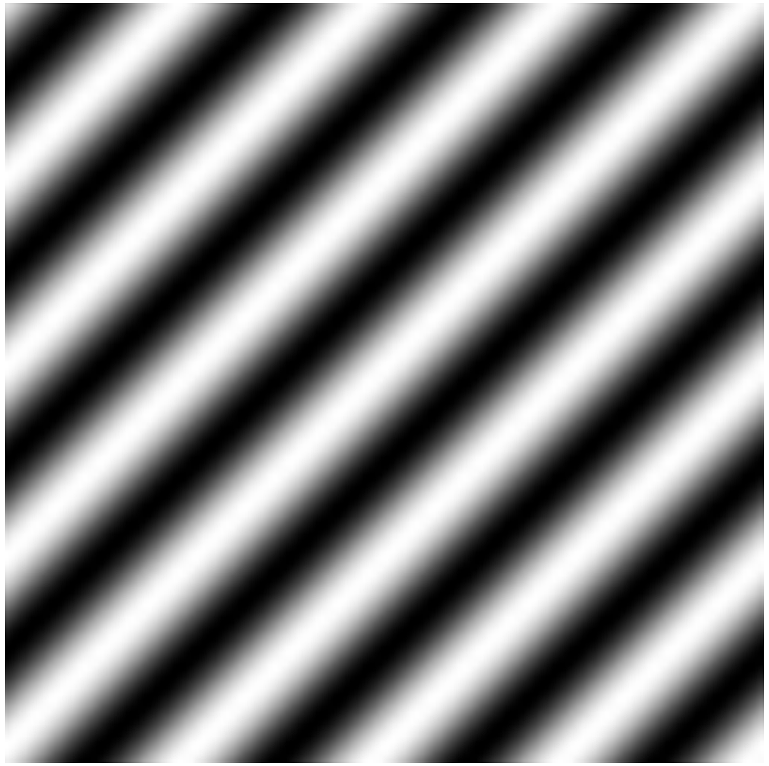


Dominio frequenze

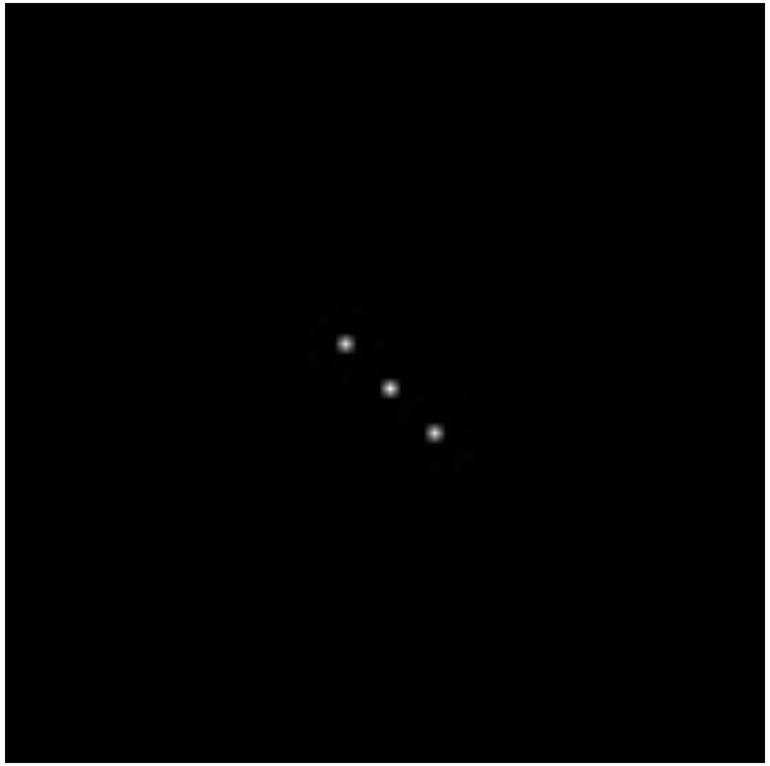


Esempio

Qual è il corrispondente di questa immagine?



Esempio



Esempio

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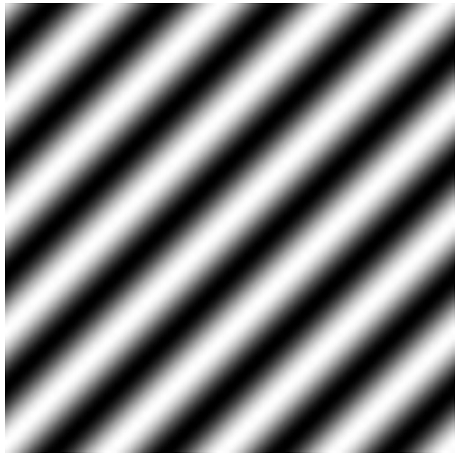

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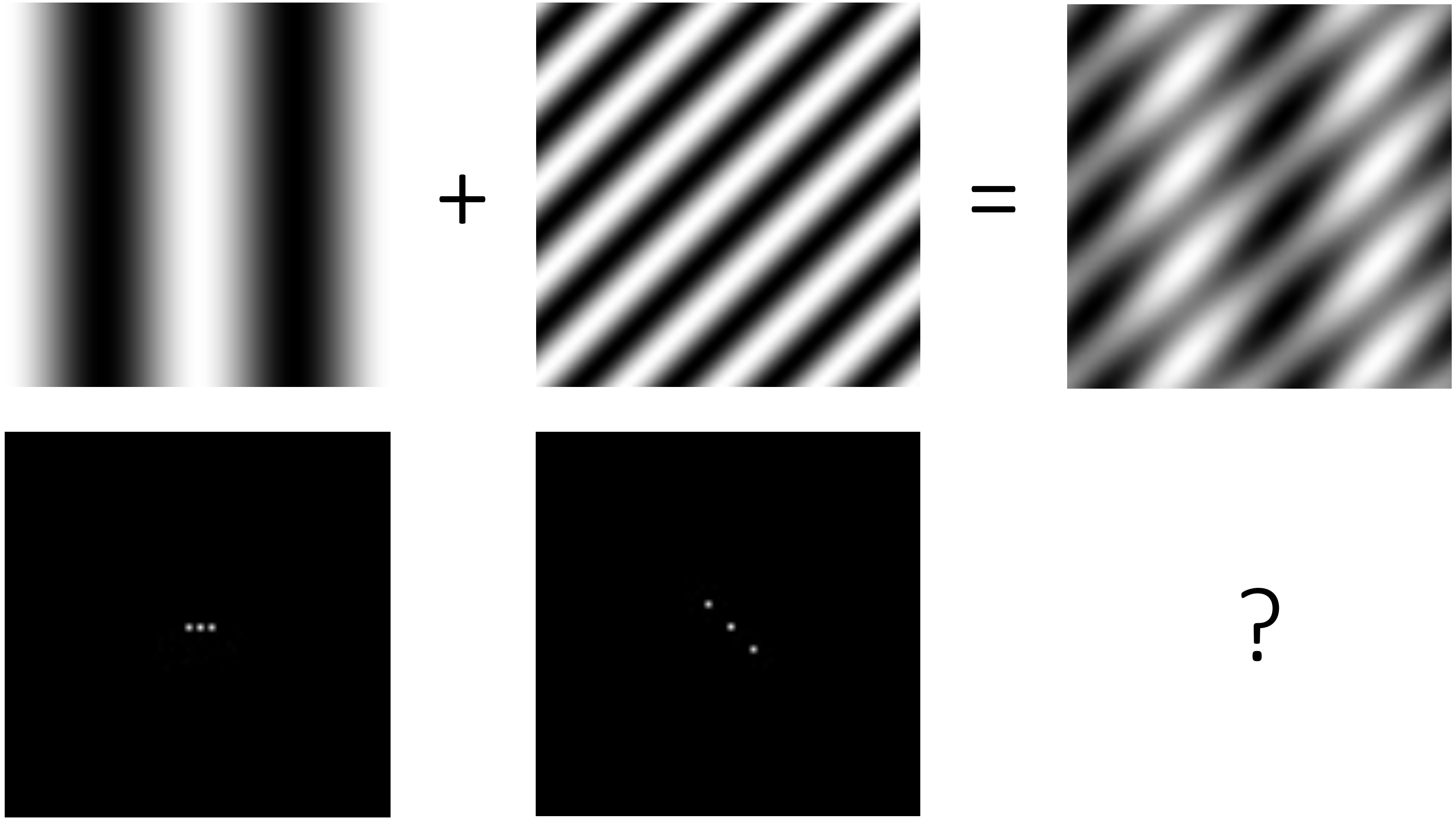
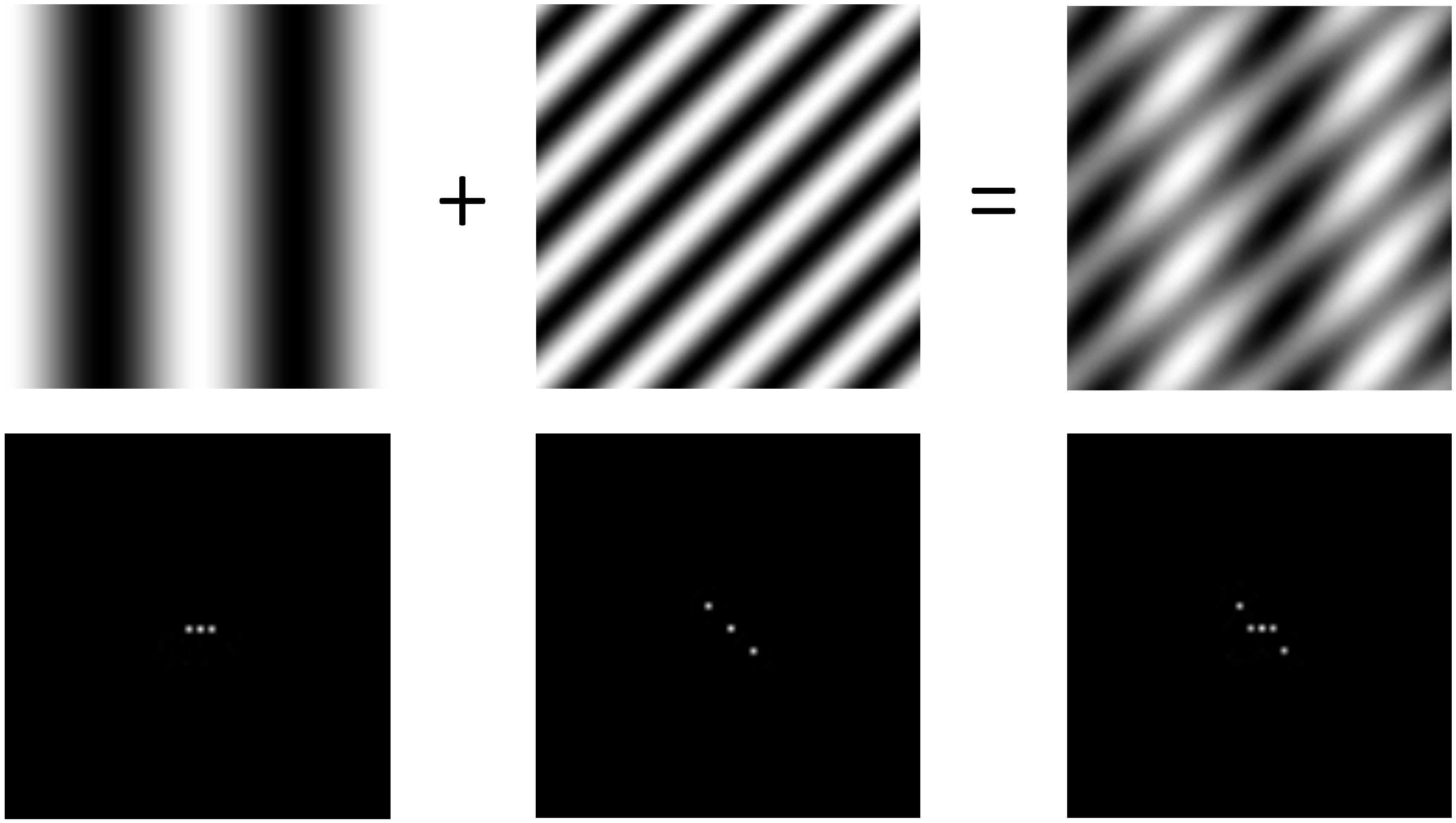
Esempio



Esempio



Trasformata di Fourier

	Diretta	Inversa
Continuo	$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi kx} dx$$	$$f(x) = \int_{-\infty}^{\infty} F(k) e^{j2\pi kx} dx$$
Discreto	$$F(k) = \sum_{k=0}^{N-1} f(x) e^{-j2\pi \frac{k}{N} x}$$ $k = 0, 1, 2, \ldots, N-1$	$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi \frac{k}{N} x}$$ $x = 0, 1, 2, \ldots, N-1$

Trasformata di Fourier

Diretta | Inversa

1D

$$F(k) = \sum_{k=0}^{N-1} f(x) e^{-j2\pi \frac{k}{N} x}$$

$k = 0, 1, 2, \ldots, N-1$

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi \frac{k}{N} x}$$

$x = 0, 1, 2, \ldots, N-1$

2D

$$F(h,k) = \sum_{h=0}^{N-1} \sum_{k=0}^{M-1} f(x,y) e^{-j2\pi \left(\frac{xh}{N} + \frac{yk}{M}\right)}$$

$h = 0,1,2, \ldots, N-1, k = 0,1,2, \ldots, M-1$

$$f(x,y) = \frac{1}{NM} \sum_{h=0}^{N-1} \sum_{k=0}^{M-1} F(h,k) e^{j2\pi \left(\frac{xh}{N} + \frac{yk}{M}\right)}$$

$\mathrm{x} = 0,1,2, \ldots, N-1, y = 0,1,2, \ldots, M-1$

Trasformata di Fourier nel dominio reale

$$\mathfrak{F}(f(x,y)) = F(h,k) = |F(h,k)|e^{-j\phi(h,k)}$$

- Ampiezza

$$|F(h,k)| = |\mathrm{R}^2(h,k) + I^2(h,k)|$$

- Fase

$$\phi(h,k) = \tan^{-1}\frac{I(h,k)}{R(h,k)}$$

La trasformata di immagini

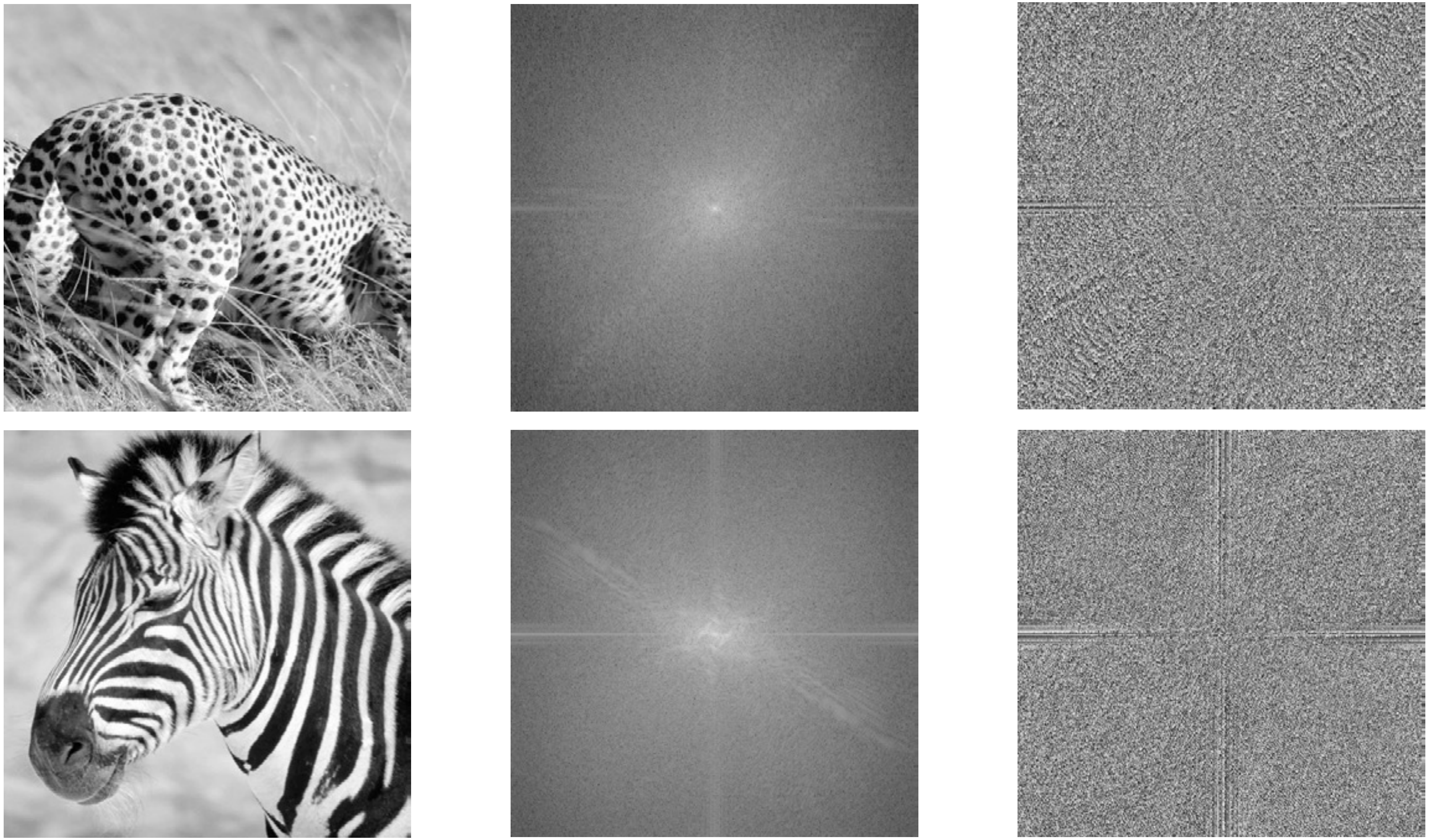


Immagine Ampiezza Fase

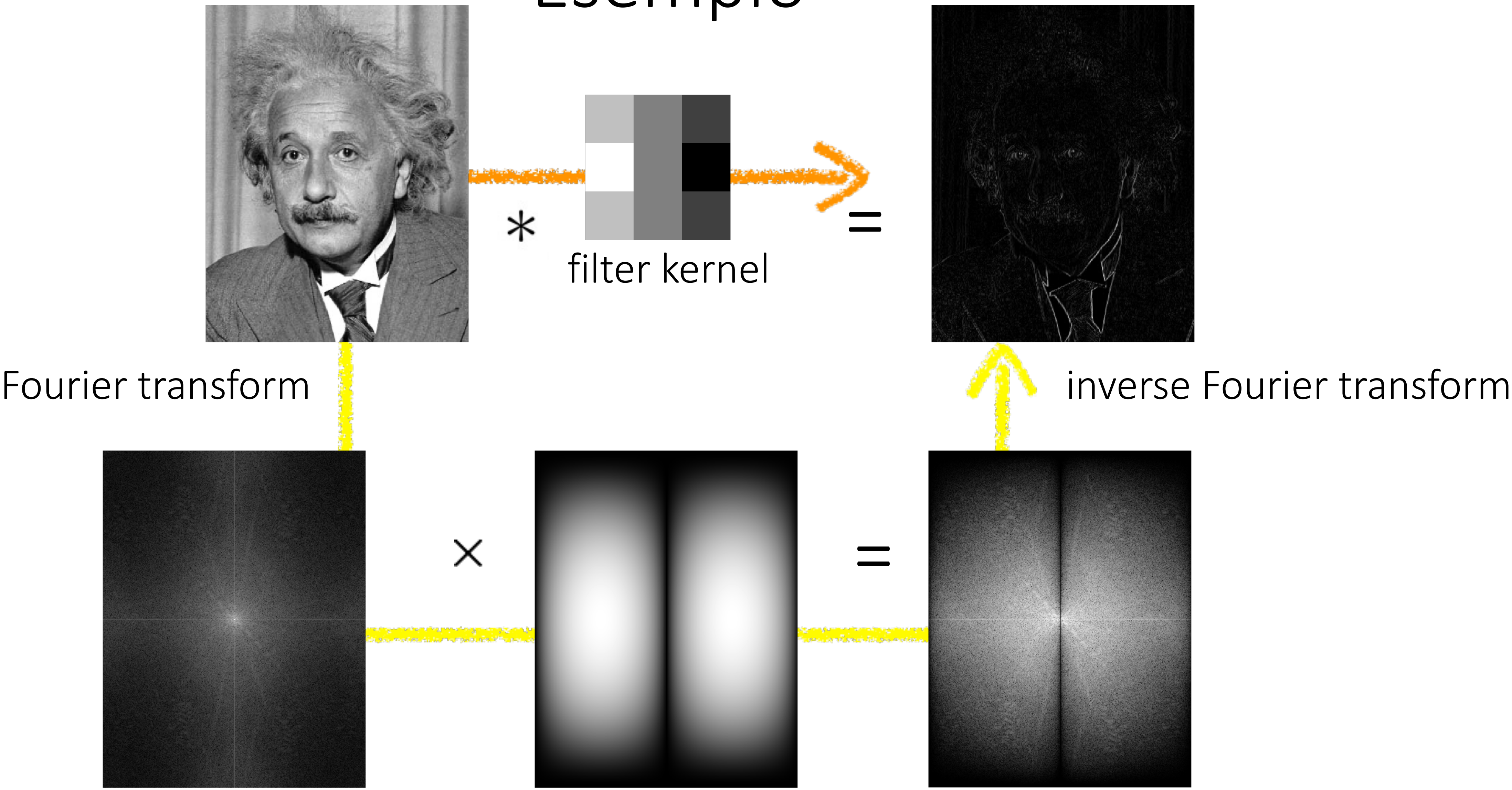
Applicazioni della FT

- Frequency-Domain Filtering
  - Teorema di convoluzione

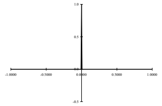
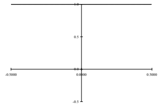
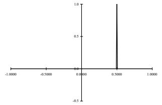
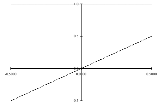
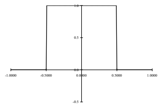
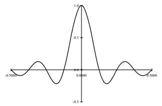
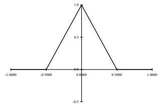
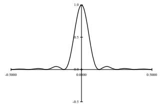
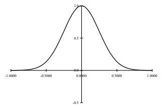
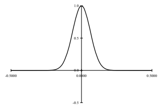
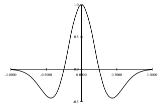
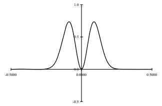
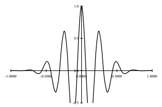
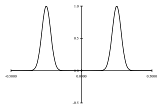
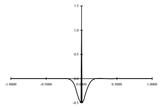
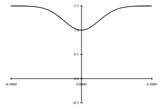
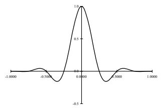
$$f(x,y) * h(x,y) = \mathfrak{F}\big(f(x,y)\big) \cdot \mathfrak{F}(h(x,y))$$

  - Conseguenza
    - Filtraggio come moltiplicazione di matrici
    - Dominio spaziale -> FT->moltiplicazione ->IFT

Esempio


*
filter kernel
=
Fourier transform
inverse Fourier transform
×
=

Alcune trasformate utili

Name		Signal		Transform	
impulse		$\delta(x)$	$\Leftrightarrow$	1	
shifted impulse		$\delta(x-u)$	$\Leftrightarrow$	$e^{-j\omega u}$	
box filter		$\text{box}(x/a)$	$\Leftrightarrow$	$a\text{sinc}(a\omega)$	
tent		$\text{tent}(x/a)$	$\Leftrightarrow$	$a\text{sinc}^2(a\omega)$	
Gaussian		$G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega;\sigma^{-1})$	
Laplacian of Gaussian		$(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2})G(x;\sigma)$	$\Leftrightarrow$	$-\frac{\sqrt{2\pi}}{\sigma}\omega^2 G(\omega;\sigma^{-1})$	
Gabor		$\cos(\omega_0 x)G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega \pm \omega_0;\sigma^{-1})$	
unsharp mask		$(1+\gamma)\delta(x)$ $-\gamma G(x;\sigma)$	$\Leftrightarrow$	$(1+\gamma)-$ $\frac{\sqrt{2\pi}\gamma}{\sigma}G(\omega;\sigma^{-1})$	
windowed sinc		$\text{rcos}(x/(aW))$ $\text{sinc}(x/a)$	$\Leftrightarrow$	(see Figure 3.29)